

THREE ESSAYS ON CONTRACTING AND CORPORATE FINANCING

BY

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DISSERTATION

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## ABSTRACT

This dissertation contains three chapters that study contracting problems associated with corporate financing. Below are the individual abstracts for each chapter.

### *Chapter 1: How Are Venture Capitalists Rewarded? The Economics of Venture Capital Partnerships*

We propose a simple model showing how investors, venture capitalists (VCs), and entrepreneurs form venture capital funds (VCFs). Investors' demand for VC services depends on their beliefs about the accuracy of VC screening and their expected revenue without the VC. The quality of screening depends on VCs' information, incentives, and expected profits. Our model characterizes equilibrium prices that VCs charge for their services and the individual payoff schedules of VCs and investors as function of project cash flows. We calibrate the model using data from existing empirical studies and find results that match the management fees charged by real-world VCs and the returns observed in the industry. Our analysis provides new insights into VC–investor partnerships and suggests that the services provided by VCs improve financing efficiency and capital formation over the business cycle.

### *Chapter 2: Credit Default Swaps, Firm Financing and the Economy*

Credit default swaps (CDSs) are thought to ease borrowing by protecting lenders against default. These contracts, however, entail a potential drawback: the “empty creditor” problem. This problem arises when creditors buy CDS insurance in excess of default renegotiation proceeds. CDS-overinsured lenders may oppose out-of-court restructuring of distressed firms, forcing them into bankruptcy even when continuation would be optimal. This paper develops a model of the demand for CDS when investment is subject to moral haz-

ard and verification is imperfect. We show that when the probability of investment success is high, CDS overinsurance allows for greater financing of firms with positive NPV projects. When investments are more likely to fail, CDS overinsurance triggers the early liquidation of firms with low continuation values, but it does not have the same effect on firms with high continuation values. The model reconciles empirical evidence showing that CDSs are most beneficial for firms that are safer and have higher continuation values. Despite the role played by CDSs in high profile bankruptcies during the financial crisis, we show that the empty creditor problem is *procyclical* (less pronounced in bad times). Our paper provides new insights on the growth of CDS markets in the early 2000s and on the optimal regulation of CDSs following the 2008–9 crisis.

### *Chapter 3: Optimal Financing with CDS Markets*

One could argue that CDSs improve risk sharing, hence credit supply and financing terms for firms. Accordingly, one would expect risky borrowers to benefit the most from CDS insurance. This is in contrast, however, with recent empirical evidence (Ashcraft and Santos (2009) and Hirtle (2009)). This paper develops a model examining optimal financing policies in the presence of CDSs when competitive lenders have different exposures to borrowers' risk. The model shows that the existence of CDSs benefits safe borrowers and harms risky ones. Following financing, the probability of a “credit event” (default following borrowers' failure to renegotiate out-of-court) is determined endogenously in a global-game setup with heterogenous payoffs. Lenders with greater risk exposure contribute to a higher probability of default and lenders that receive a higher weight in the financing process are more influential in determining renegotiation outcomes. Prior to financing, lenders with

higher risk exposure benefit the most from CDS insurance and allow for reduced repayments — increasing borrowers' surplus in the absence of distress. Riskier (safer) borrowers finance more heavily with lenders that have lower (higher) exposure; lenders, in turn, insure less (more) often and the probability of default given distress is lower (higher). The model sheds light on the liquidity of CDSs across risk-differentiated borrowers and allows for proposals that improve the welfare of market participants.

*To Elisa, Sophia, My Parents, and My Sisters*

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# Chapter 1

## How Are Venture Capitalists Rewarded? The Economics of Venture Capital Partnerships

### 1.1 Introduction

Venture capital financing is a fast-growing segment of the capital markets landscape. In 2008 alone, venture-backed companies accounted for 12 million jobs in the U.S. (11% of all private sector employment) and nearly \$3 trillion in gross revenues (21% of the GDP).<sup>1</sup> Despite their importance, relatively little is known about how venture capital funds emerge, how they set their payoff structures, and whether they contribute to the overall efficiency of capital allocation. Recent empirical studies provide key insights into the world of venture capitalists (e.g., Kaplan and Schoar (2005), Puri and Zarutskie (2008), and Metrick and Yasuda (2010)), but theoretical research lags in providing a framework that rationalizes and integrates existing findings on venture capital activity.

VC-financing often brings together young entrepreneurial firms and investors with limited

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<sup>1</sup>See *Global Insight* (2009) for additional statistics about the venture capital industry.



project-specific expertise. Rather than investing directly in those firms, investors form partnerships with agents believed to have some expertise (venture capitalists) through venture capital funds (Sahlman (1990)).<sup>2</sup> VCs screen companies and report on the value potential and progress of multiple ventures. A typical VCF makes investments in several stages, with ventures often abandoned when initial returns are low (Barry (1994) and Puri and Zarutskie (2008)). During the life of the VCF, the VC commonly contributes to a small fraction of the fund (around 1%) and receives an annual management fee of 1.5% to 2.5% of the committed capital (Kaplan and Schoar (2005) and Metrick and Yasuda (2010)). In addition to the fixed compensation, the VC receives a variable portion that depends on the realized profit of the investment. It has become practice to reward VCs with 20% of the profits of successful ventures (“carried interest”) through the IPO process (see Gompers and Lerner (1999)).

While relatively well documented, many questions arise about the set of economic relations and payoffs that characterize venture capital partnerships. For example, under what conditions are investors willing to pay for the services VCs provide? How accurate is the information contained in the reports VCs give to investors? What explains the observed reward structures in the VC industry? Can VCF arrangements contribute to capital market efficiency?

This paper tackles those questions by characterizing conditions under which VC–investor partnerships emerge. VCs are special to the extent that they *may* be in a better position to evaluate project quality and determine whether entrepreneurs are knowledgeable about

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<sup>2</sup>According to the National Venture Capital Association, VCs tend to specialize in industries in which they have years of prior experience (VCs are oftentimes former entrepreneurs in those industries). See also Bottazzi et al. (2008).

their projects. Because VCs' information can increase expected gains from project funding, investors are willing to pay for screening so long as VC reports are sufficiently accurate. In exchange, VCs charge investors part of the economic surplus generated by screening. We formalize equilibrium conditions under which VCs increase the probability that good projects are financed and reduce the probability that bad projects receive funding. We also describe the payoff schedules of VCs and investors under the VCF partnership. To our knowledge, this is the first study to model interactions between investors, VCs, and entrepreneurs in a single framework. It is also the first study to jointly characterize the conditions for emergence of VCFs, the payoffs of the various participants of VCFs, and the efficiency of VC financing.

Let us discuss the intuition underlying our model. We consider a setting in which investors are uninformed about the quality of any specific projects, while entrepreneurs may be either informed or uninformed about project quality. Investors can either finance entrepreneurs directly (uninformed financing) or pay VCs for project screening. Under direct financing, investors may charge a uniform price. If investors offer entrepreneurs contracts requiring high returns, only informed entrepreneurs with good projects will accept the offer. If investors require low returns, then entrepreneurs of all types will accept direct financing. If it is optimal to demand high returns, uninformed entrepreneurs with good projects will not be financed. If it is optimal to demand low returns, then bad projects will be financed and investors lose rents from informed entrepreneurs with good projects.

Under indirect financing, VCs may increase investors' expected payoffs by screening entrepreneurs and helping identify good projects. In particular, the reports provided by VCs may help synchronize the beliefs of investors and entrepreneurs about project quality. This

increases the willingness of entrepreneurs with good projects to accept contracts that demand high returns, increasing investors' expected payoffs. Investors pay for VC services to the extent that reports reflect information gleaned through the screening process. Importantly, however, VC reports may be *less than accurate*, depending on VCs' incentives and knowledge about projects and entrepreneurs.

In solving this problem, we use the concept of a Perfect Bayesian Equilibrium. Under this concept, sequential rationality implies that VC reports maximize the probability of higher rounds of financing so as to generate higher levels of compensation through carried interest. Accordingly, on the margin, VCs might want to produce optimistic reports on the projects in their portfolios, even when those projects are relatively unattractive from investors' perspective. Importantly, however, VCs only receive the carried interest after the initial rounds of financing prove to be successful. As such, if VCs believe that the pool of projects under consideration is of high quality, the gains associated with misreporting are small. On the flip side, if VCs believe that projects are of intrinsically poor quality, they gain little by misreporting, since in expectation their carried interest will be low (poor projects are unlikely to survive much beyond the initial financing rounds).

If VCs are uncertain about project quality, however, they have incentives to produce inaccurate reports; reports that might lead to overinvestment. Intuitively, this could undermine the value of VCs as intermediaries, hence the emergence of VCFs. Nonetheless, even though VCs may misreport in equilibrium, we show that distortions caused by inaccurate reporting might still let investors and entrepreneurs synchronize their beliefs about project quality. The key for this result is the degree of correlation between the information

of VCs and entrepreneurs, which may allow for valuable data gathering even when reports are imperfect. We show that this correlation allows for surplus extraction from uninformed entrepreneurs with high-quality projects and increases investors' expected profits under the VC–investor partnership. VCFs emerge as optimal arrangements in this environment despite imperfections in information and incentives.

Our VCF model provides interesting insights about the role VCs play in economic cycles. When the fraction of good projects in the economy is small (“busts”), the proportion of uninformed entrepreneurs with good projects is reduced. Accordingly, under direct financing, investors are likely to demand high returns from entrepreneurs. This translates into financing inefficiencies, since some good projects do not receive funding. When the probability of good projects is high (“booms”), investors may demand relatively lower returns. This result is also inefficient since bad projects may receive funding. Under VCF financing, in contrast, uninformed entrepreneurs with good projects as well as those with bad projects adjust their beliefs about project quality. In booms, for example, the former accept contracts with higher promised returns, while the latter reject most contracts. Notably, this outcome does not require VCs to be fully-informed agents. Indeed, among other results, our model shows that VCFs contribute to efficiency in capital formation in booms even in the absence of strong assumptions about VC information.

The empirical counterpart for the price of screening in our model is the management fee charged by VCs on the committed capital of the VCF. We use results from existing empirical studies (e.g., Sahlman (1990), Phillips and Kirchoff (1989), and Puri and Zarutskie (2008)) and the restrictions of the model to calibrate relevant parameters. For example, we estimate

management fees to be 16.4% of committed capital. This number is very similar to the fee of 16.1% reported by Metrick and Yasuda (2010). That estimate is also well within the 15–20% range of Gompers and Lerner (1999). Following our model, we also calculate the VCFs’ annual rate of return. Our estimate of 22% per year virtually matches the figures reported by Sahlman (1990) and Kaplan and Schoar (2005), which equal 21% and 20%, respectively.

We also characterize the VC compensation structure. In doing so, our model incorporates features from the most common payment schedules of the industry, such as those described in Metrick and Yasuda (2010). We show that investors receive the exit proceeds from the VCF until they get repaid the full carry basis. After that, both investors and VCs share the VCF’s cash flows. Our model also describes an interesting tension between management fees and carried interest. These forms of compensation oppositely depend on the determinants of VCs’ opportunity costs of investing. The relations described by the model are consistent with Gompers and Lerner (1999), who show that management fees (carried interests) are negatively (positively) related to proxies for the experience and skills of VCs. We estimate the carried interest to be 4.4% of committed capital for a carry level of 20%, which is somewhat short of the 7.3% bound of Metrick and Yasuda. Given the dynamics described by our model, however, a carried interest of 7.3% would be associated with a management fee of 12.9%. This number is consistent with the 12.1% figure estimated by Metrick and Yasuda for a fee of 1.5% of committed capital.

Most theoretical papers in the literature focus on the interplay between the VC and the entrepreneur. Some consider the VC to be an investor with skills to screen projects (Bernhardt and Krassa (2008)) or an insider that signals to outside investors whether the project

is sound (Chan (1983) and Admati and Pfleiderer (1994)). Other studies view the VC as a principal whose role is to ensure that entrepreneurs choose the optimal level of effort (Amis et al. (1990)). In these papers, the notion of “investor” is reduced to an exogenous association of VCs and financiers, which oversimplifies the characterization of the venture capital industry. As a result of their setup, those models cannot explain the emergence of VCFs and are unable to characterize the fees charged by VCs.

The paper that is perhaps closest to ours is Axelson et al. (2008). Those authors consider a model with three agents: investors, venture capitalists, and fly-by-night operators. One of their central results is that venture capital partnerships use a mix of ex-post and ex-ante financing. The compensation scheme the authors derive resembles that of a debt-like contract: for outcomes lower than the amount invested, investors seize everything; while for outcomes above that threshold, investors and VCs receive a fraction of the project’s profits. In the Axelson et al. model, VCs are assumed to be fully informed about the quality of projects, while investors and fly-by-night operators are uninformed. Notably, their analysis does not explain what determines the management fee charged by VCs. Our framework, in contrast, allows for a more general information structure; for example, we do not impose that either entrepreneurs or VCs are fully informed. In addition, we focus on the payoffs that VCs and investors derive from the VCF. By explicitly modeling entrepreneurs, the payoff structures of VCs and investors, and the correlation of information between VCs and entrepreneurs, our analysis extends the existing theoretical literature into new directions.

The debate on the appropriate level of management fees charged by VCs has intensified

since the financial crisis.<sup>3</sup> Because these fees are a significant portion of VCs’ compensation and are invariant to the performance of the VCFs, the fixed compensation structure has come under scrutiny after the drop in industry returns in late 2007. Investors are calling for a “realignment of interests,” increased transparency in reporting, and demanding management fees to reflect the quality of the portfolios VCs oversee. While those demands for reform may be warranted, it is difficult to evaluate their merit in the absence of a model that rationalizes the payoff structures of the VC industry. Our analysis helps understand the implications of these kinds of debates by formalizing the economics of the rewards currently observed in the industry.

The remainder of the paper is organized as follows. Section 2 describes the model setup. In Section 3, we solve the model and examine its main results. Section 4 performs calibrations examining the main results of our model. Section 5 presents a list of testable empirical implications. Section 6 concludes the paper. All proofs are in Appendix A.

## 1.2 The Model

We set out to frame our model according to stylized facts of the industry. Entrepreneurs are endowed with projects and entrepreneurial capital, which can be thought as human capital, talent, ideas, and inventions. VCs are special in that they may have relevant information about entrepreneurs’ projects. This is in accordance with the Venture Census 2008 (National Venture Capital Association (2008)), which reports that VCs are generally former entrepreneurs with industry-specific expertise. Entrepreneurs’ projects require en-

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<sup>3</sup>See Prequin (2007, 2009) and Wyatt (2009) for industry reports on the intensity of this debate.

entrepreneurial capital and outside finance, which is provided by investors. Ample evidence suggests that a VCF invests in several rounds and that subsequent financing happens only if the previous round of investment is considered successful (see Sahlman (1990), Barry (1994), and Puri and Zarutskie (2008)). To capture these features, we assume that financing takes place in multiple periods.

### 1.2.1 Players and Environment

There are three periods  $\{0, 1, 2\}$ , financing takes place in periods 0 and 1, and there is no discounting. The economy has an investor  $K$ , an entrepreneur  $E$ , and a venture capitalist  $V$ . The investor has an amount  $d$  that he can lend to the entrepreneur. The entrepreneur is penniless, but is endowed with entrepreneurial capital and a project. Both the entrepreneur and the VC hold private information about the quality of the entrepreneur's project. The information of the entrepreneur and the VC can be correlated. All agents are risk-neutral.

The entrepreneur is endowed with entrepreneurial capital  $m_H$ , which is assumed to be common knowledge. Projects can be either good ( $G$ ) or bad ( $B$ ). A project is denoted by  $s \in \{G, B\}$ , and its outcome by  $\pi(s) \in \{\pi_L, \pi_H\}$ . We assume  $\pi_H > 1$  and  $\pi_L = 0$ . Bad projects always have an outcome of  $\pi(B) = \pi_L$ .

Projects generate outcomes after being implemented. The investment technology in period 0 dictates that projects require  $m_H$  units of entrepreneurial capital and  $1 - m_H$  units of external funding, where  $0 \leq 1 - d \leq m_H < 1$ . In period 1, the investment technology requires projects to have one unit of outside finance. The entrepreneur and the VC face opportunity costs of  $m_H + \bar{u}$  and  $\bar{u}$ , respectively, for implementing a project. The parameter  $\bar{u} > 0$  captures



general characteristics of the industry in which the entrepreneur and the VC operate.<sup>4</sup>

The entrepreneur and the VC can be informed or uninformed. If informed, they know the project's quality, otherwise they do not know if the project is good or bad. The types of the entrepreneur and the VC belong to the set  $T = \{\{G\}, \{B\}, \{U\}\}$ , with  $\{U\} = \{G, B\}$ ; they are denoted by  $\iota_E$  and  $\iota_V$ , respectively. These types are private information. We denote the probability of a good project by  $\lambda \in (0, 1)$ , the probability of a good outcome by  $p \in (0, 1)$ , and the probability of being informed about the project's quality by  $q \in (0, 1)$ . To make the model interesting, we assume it is always profitable to finance a project that is known to be good, that is,  $p\pi_H - 1 - u > 0$ . To simplify the exposition, we assume  $\pi_H - \frac{m_H + \bar{u}}{\lambda p} > 1$ . As will become clear in the equilibrium analysis, this assumption implies that a project will be financed in the second period if it succeeds in the first period.

Given that the VC is informed, the probability that the entrepreneur is informed is  $\theta$ . Conversely, given that the VC is uninformed, the probability that the entrepreneur is uninformed is  $\gamma$ . Consistency of probabilities require that  $q(1 - \theta) = (1 - q)(1 - \gamma)$ . For  $i, j \in \{E, V\}$  we define  $\mu(\iota_i, \iota_j)$  as the prior distribution of types,  $\mu(\iota_i)$  as the marginal distribution, and  $\mu(\iota_i | \iota_j)$  as the conditional distribution.

It is important to frame the information (or “expertise”) that VCs have about the projects they screen and about the entrepreneurs behind those projects. The role of VC expertise and experience has been studied by recent literature looking at VC characteristics and the

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<sup>4</sup>In a more general setting, the opportunity costs of the entrepreneur and the VC could be given by  $o_E(\cdot)$  and  $o_V(\cdot)$ , respectively. Both the entrepreneur's and the VC's opportunity costs would be a function of  $\bar{u}$ . In addition, the entrepreneur's opportunity cost  $o_E$  would depend on his entrepreneurial capital. In order to keep the model parsimonious, we set  $o_E = m_H + \bar{u}$  and  $o_V = \bar{u}$ . In this way, we reduce the number of free parameters in our calibration exercise and are able to assess the power of the model in replicating observed VC compensation structures.

performance of the projects in their portfolios (e.g., Gompers et al. (2005) and Bottazzi et al. (2008)). We consider the following possibilities for VC expertise:<sup>5</sup>

(1) The VC has *industry expertise* and *knowledge about entrepreneur's type*; or  $(\theta, \gamma) = (1, 1)$ . This is to say that the VC gauges the type of the entrepreneur based on his own experience with the sort of project he is screening. For example, when considering a standard, simple (new, complex) project, the VC anticipates the entrepreneur may know as much (as little) as he does about the project's likely outcome. Simply put, the VC and the entrepreneur have highly correlated information about the project; and

(2) The VCs has only *industry expertise*; or  $(\theta, \gamma) = (q, 1 - q)$ . This is to say that the VC might know the project's type, but the VC does not know about the entrepreneur's specific knowledge about the project. For example, when he knows the project's type, the VC can only assign a general probability  $q$  that the entrepreneur also knows the project's type.

### 1.2.2 Timing, Strategies, and Payoffs

The timing of the model is described in Figure 1.1. In period 0, Nature chooses the types of the entrepreneur and the VC. The uninformed investor decides if he wants to finance the entrepreneur directly or to use the VC as an intermediary. If he chooses the direct route, the investor sends a contract to the entrepreneur. If the investor chooses the intermediated route, he sends the VC a contract featuring a fixed price for a report containing information about the project. Upon receiving the report, the investor decides whether to fund the

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<sup>5</sup>Essentially, the state space is  $\{G, B\}$  and the types of the entrepreneur and the VC represent their information regarding the realized state. We restrict our attention to polar cases for two reasons: (i) a more general correlation structure would make the number of parameters to be pinned down by any calibration exercise greater than the number of constraints generated by the model, and (ii) the results provided by a more general information structure are qualitatively similar.

project. Let us detail the events that take place in each period of the model.

[Figure 1.1 about here]

### • Period 0

Nature chooses the type of the project and the knowledge of both the entrepreneur and the VC. A financing contract specifies a nonnegative price  $R$  that needs to be paid to the investor and the ownership over the project in period 1. We assume limited liability such that  $R(\pi_s) \leq \pi_s$ . Therefore,  $R(\pi_L) = 0$  and the relevant choice is about the price in the event the project succeeds  $R \equiv R(\pi_H)$ . Since projects do not require entrepreneurial capital in period 1 and the VC has a lower opportunity cost for implementing a project in that period, the investor will propose a contract in which he has ownership of the project.

The investor decides either to finance the entrepreneur directly or to hire a VC to learn about the venture. The VC has belief  $\mu(\iota_E|\iota_V)$  that the entrepreneur is of type  $\iota_E$ , and chooses a nonnegative price  $z_{\iota_V}$  for his information. After observing  $z$  and forming belief  $\mu_K(\iota_V|z) \in [0, 1]$  that the VC is of type  $\iota_V$ , the investor takes action  $k(z) \in \{0, 1\}$ , where  $k = 0$  if the investor rejects the price, and  $k = 1$  if the investor accepts the price. If the investor chooses uninformed financing, he sends the entrepreneur a contract with nonnegative price  $R_0^U(z)$ . The entrepreneur of type  $\iota_E$  holds a belief  $\mu_E(\iota_V|\iota_E, z)$  that the VC is of type  $\iota_V$ , and takes action  $e_{\iota_E}^U(R_0^U) \in \{0, 1\}$ , where  $e_{\iota_E}^U = 0$  if he rejects the contract, and  $e_{\iota_E}^U = 1$  if he accepts.

If the investor chooses informed financing, he signs a contract with the VC. The VC of type  $\iota_V$  selects information  $\tau_{\iota_V} \in T$  to reveal. The information  $\{G\}$  implies that the VC

is informed that the entrepreneur has a good project,  $\{B\}$  implies the VC is informed that the entrepreneur has a bad project, and  $\{U\}$  implies that the VC is uninformed about the quality of the project. Upon receiving a report, the investor has belief  $\mu_K(\iota_V|z, \tau) \in [0, 1]$  that the VC is of type  $\iota_V$ , and chooses a contract with a nonnegative price  $R_0^I(\tau)$ . The entrepreneur of type  $\iota_E$  forms belief  $\mu_E(\iota_V|\iota_E, z, \tau)$  that the VC is of type  $\iota_V$  and takes action  $e_{\iota_E}^I(R_0^I) \in \{0, 1\}$ , where  $e_{\iota_E}^I = 0$  if he refuses to sign the contract, and  $e_{\iota_E}^I = 1$  if he decides to sign it. If he chooses the former, the game ends. For convenience, we assume that the investor, the entrepreneur, and the VC accept proposals whenever they are indifferent.

Two technical aspects of our modeling choices are worth discussing. First, we assume that the investor cannot commit to a rule following a report. In this way, our analysis does not reduce to a mechanism design problem and it cannot be used to find optimal contracts. This assumption is consistent with investors' right not to invest beyond the initial committed capital. It implies the following *ratchet effects*: (1) the investor cannot commit to invest if he is told that the investment under consideration has negative NPV, and (2) the investor cannot refrain from demanding high returns if he is told that the project is good.

Second, under informed financing, it is the investor that offers the entrepreneur a contract. While this simplifies the model set up and analysis, it will be clear from the equilibrium solution that our assumption is made without loss of generality. Indeed, we can show that the outcome that obtains when the design of contracts is delegated to the VC is also an equilibrium outcome of our model. Our simpler set up, however, is useful in helping one to gauge the role of VC reports in coordinating the expectations of investors and entrepreneurs

in a way that benefits not only the private interests of VCs, but also the VCF.<sup>6</sup>

• **Period 1**

Payments are made according to project outcomes and contracts in place. The payoff of the entrepreneur of type  $\iota_E$  is:

$$\begin{aligned} u_{\iota_E}^0 &\equiv (1 - k) [e_{\iota_E}^U \mathbf{1}_{\{s=G\}} p (\pi_H - R_0^U) + (1 - e_{\iota_E}^U) (m_H + \bar{u})] + \\ &k [e_{\iota_E}^I \mathbf{1}_{\{s=G\}} p (\pi_H - R_0^I) + (1 - e_{\iota_E}^I) (m_H + \bar{u})]. \end{aligned} \quad (1.1)$$

The payoff of the investor is:

$$\begin{aligned} u_K^0 &\equiv (1 - k) [e_{\iota_E}^U (\mathbf{1}_{\{s=G\}} p R_0^U - (1 - m_H))] + \\ &k [e_{\iota_E}^I (\mathbf{1}_{\{s=G\}} p R_0^I - (1 - m_H) - z_{\iota_V}) - (1 - e_{\iota_E}^I) z_{\iota_V}]. \end{aligned} \quad (1.2)$$

The payoff of the VC of type  $\iota_V$  is:

$$u_{\iota_V}^0 \equiv k z_{\iota_V}. \quad (1.3)$$

If  $\pi_L = 0$ , there are insufficient funds for a new financing round and the game ends. If the outcome is  $\pi_H$ , all players believe that the project is good. If the investor chose uninformed financing in period 0, he offers a contract with a nonnegative price  $R_1^U$  to the VC. The VC chooses  $v_{\iota_V}^U (R_1^U) \in \{0, 1\}$ , where  $v_{\iota_V}^U = 0$  if he rejects the contract; otherwise,  $v_{\iota_V}^U = 1$ . If the investor chose informed financing, he offers the VC a contract to implement the entrepreneur's project in period 1 with a nonnegative price  $R_1^I$ . The VC of type  $\iota_V$  takes

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<sup>6</sup>If the VC offered the contract, we would have a delegation game. The outcome of the selected equilibria in our model maximizes the carried interest paid to the VC, and this is the *same outcome* that would be observed in a delegation game. This assumption is made without loss of generality, but greatly simplifies the analysis.

action  $v_{\iota_V}^I (R_1^I) \in \{0, 1\}$ , where  $v_{\iota_V}^I = 0$  if he rejects the contract, and  $v_{\iota_V}^I = 1$  if he accepts it. If the contract is accepted, investment is made.

• **Period 2**

The payoffs of the second period are realized and the game ends. The payoff of the investor is:

$$u_K^1 = (1 - k) e_{\iota_E}^U \mathbf{1}_{\{\pi_s = \pi_H\}} v_{\iota_V}^U (pR_1^U - 1) + k e_{\iota_E}^I \mathbf{1}_{\{\pi_s = \pi_H\}} v_{\iota_V}^I (pR_1^I - 1). \quad (1.4)$$

Finally, the payoff of the VC of type  $\iota_V$  is:

$$\begin{aligned} u_{\iota_V}^1 = & (1 - k) e_{\iota_E}^U \mathbf{1}_{\{\pi_s = \pi_H\}} [v_{\iota_V}^U p (\pi_H - R_1^U) + (1 - v_{\iota_V}^U) \bar{u}] + \\ & k e_{\iota_E}^I \mathbf{1}_{\{\pi_s = \pi_H\}} [v_{\iota_V}^I p (\pi_H - R_1^I) + (1 - v_{\iota_V}^I) \bar{u}]. \end{aligned} \quad (1.5)$$

Figure 1.2 describes the investor's and VC's proceeds as a function of the VCF's cash flow. When the investor chooses to offer a contract to the VC, a VCF emerges. The entrepreneur is financed and the investor receives  $R_0^I$  if the project succeeds. This is the first cash flow of the VCF. In the second round of financing, the VC implements the entrepreneur's project and the outcome in the event of success is given by  $\pi_H$ . This is the VCF's second cash flow and is shared between the VC and the investor, who receive  $\frac{\pi_H - R_1^I}{\pi_H}$  and  $\frac{R_1^I}{\pi_H}$ , respectively.

[Figure 1.2 about here]

## 1.3 Equilibrium

### 1.3.1 Preliminaries

The equilibrium concept we use is that of a Perfect Bayesian Equilibrium (PBE). We restrict ourselves to pure strategies. Let  $S(z) \subset T$  be the set of types of VC that charge  $z$ . The beliefs of the investor and the entrepreneur after observing  $z$  are consistent according to Bayes's rule if  $\mu_K(\iota_V|z) = \frac{\mu(\iota_V)}{\sum_{\iota'_V \in S(z)} \mu(\iota'_V)}$  and  $\mu_E(\iota_V|\iota_E, z) = \frac{\mu(\iota_V|\iota_E)}{\sum_{\iota'_V \in S(z)} \mu(\iota'_V|\iota_E)}$  for  $\iota_V \in S(z)$ , and  $\mu_K(\iota_V|z) = \mu_E(\iota_V|z) = 0$  for  $\iota_{E_2} \notin S(z)$ .

Analogously, if  $S(\tau) \subset T$  is the set of types of VC that report  $\tau$ , the beliefs of the investor and the entrepreneur after observing  $\tau$  are *Bayes-consistent* if  $\mu_K(\iota_V|z, \tau) = \frac{\mu_K(\iota_V|z)}{\sum_{\iota'_V \in S(\tau)} \mu(\iota'_V|z)}$  and  $\mu_E(\iota_V|\iota_E, z, \tau) = \frac{\mu_E(\iota_V|\iota_E, z)}{\sum_{\iota'_V \in S(\tau)} \mu(\iota'_V|\iota_E, z)}$  for  $\iota_V \in S(\tau)$ , and  $\mu_K(\iota_V|z, \tau) = \mu_E(\iota_V|\iota_E, z, \tau) = 0$  for  $\iota_V \notin S(\tau)$ . Given our assumptions about the correlation of types, the investor's beliefs about the type of the entrepreneur are directly obtained from his beliefs regarding the type of the VC  $\mu_K(\iota_E|z) = \sum_{\iota'_V \in S(z)} \mu_K(\iota'_V|z) \mu(\iota_E|\iota'_V)$  and  $\mu_K(\iota_E|z, \tau) = \sum_{\iota'_V \in S(\tau)} \mu_K(\iota'_V|z, \tau) \mu(\iota_E|\iota'_V)$ .

**Definition 1** *A collection of strategies*

$$((k, R_0^U, R_0^I, R_1^U, R_1^I), (e_{\iota_E}^U, e_{\iota_E}^I), (z_{\iota_V}, \tau_{\iota_V}, v_{\iota_V}^U, v_{\iota_V}^I))$$

and beliefs

$$\left( (\mu_K(\iota_i|z), \mu_K(\iota_i|z, \tau))_{i \in \{E, V\}}, (\mu_E(\iota_V|\iota_E, z), \mu_E(\iota_V|\iota_E, z, \tau)), \mu(\iota_E|\iota_V) \right)$$

constitute a (pure strategy) PBE if:

- (i) For every history of actions, strategies maximize expected payoffs given beliefs; and
- (ii) Beliefs are updated using Bayes' rule whenever possible.

Our first lemma will be useful in order to establish later results. It states that the optimal strategies of the investor and the VC regarding refinancing are the same under uninformed

and informed financing.

**Lemma 1** *In any PBE  $R_1^U = R_1^I = R_1 = \pi_H - \frac{\bar{u}}{p}$ ,  
 $v_{\iota_V}^U(R) = v_{\iota_V}^I(R) = \begin{cases} 1, & \text{if } 0 \leq R \leq \pi_H - \frac{\bar{u}}{p} \\ 0, & \text{if } R > \pi_H - \frac{\bar{u}}{p} \end{cases} \quad \forall \iota_V \in T \setminus \{B\}.$*

To characterize the outcome of markets under uninformed financing and under VC financing, we first need to define equilibria in which VCFs emerge.

**Definition 2** *A PBE is a VC-equilibrium if  $k = 1$  and  $z_{\iota_V} > 0 \quad \forall \iota_V \in T$ ; i.e., if the investor chooses informed financing and pays a positive fee to the VC.*

We now proceed to show that there is no separating equilibria with respect to prices; i.e., equilibria where different types of VCs charge different prices for their information. We then show that the investor can restrict attention to charging either a high price or a low price under uninformed financing. The former will be accepted only by informed entrepreneurs with good projects, whereas the latter will be accepted by all entrepreneurs except those informed with bad projects. This will allow the derivation of the expected payoff of the investor given his strategy so that it can be later compared with the expected payoff under informed financing.

**Lemma 2** *There is no VC-equilibrium where VCs charge different prices.*

Although this result is consistent with empirical evidence (e.g., Sahlman (1990) and Gompers and Lerner (1999)), it is still viewed in the literature as a puzzle (as discussed by Kaplan and Schoar (2005)). If VCs charged prices contingent on their information, that would identify their types and allow the investor to get the information he wanted at no cost. This implies a flat price and the possibility that VCs receive compensation even when



they are uninformed about the quality of projects. As a consequence of this lemma, we will focus our attention to the cases in which  $S(z) = T$ .

The next lemma derives the optimal choices of the entrepreneur under informed and uninformed financing. In essence, it states that the entrepreneur's decision regarding the contract is dictated by a set of cutoff rules.

**Lemma 3** *Let  $\underline{R}_0^U = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ ,  $\overline{R}_0^U = \pi_H - \frac{m_H + \bar{u}}{p}$ ,  $\overline{R}_0^I = \pi_H - \frac{m_H + \bar{u}}{p}$ , and  $\underline{R}_0^I(\tau) = \pi_H - \frac{m_H + \bar{u}}{\mu_E(G|\{U\}, z, \tau)p}$  for  $\mu_E(G|\{U\}, z, \tau) > 0$ . In any PBE equilibrium with  $S(z) = T$ :*

- (i)  $e_{\{B\}}^k(R_0^i) = 0 \ \forall R_0^i \in \mathbb{R}^+$ ,
- (ii)  $e_{\{U\}}^i(R_0^i) = \begin{cases} 0, & \text{if } \mu_E(G|\{U\}, z, \tau) = 0 \\ 1, & \text{if } 0 \leq R_0^i \leq \underline{R}_0^i \text{ and } \mu_E(G|\{U\}, z, \tau) > 0, \text{ and} \\ 0, & \text{if } R_0^i > \underline{R}_0^i \text{ and } \mu_{E_1}(G|\{U\}, z, \tau) > 0 \end{cases}$
- (iii)  $e_{\{G\}}^i(R_0^i) = \begin{cases} 1, & \text{if } 0 \leq R_0^i \leq \overline{R}_0^i \\ 0, & \text{if } R_0^i > \overline{R}_0^i \end{cases} \text{ for } i = U, I.$

According to the last lemma, if the entrepreneur is informed that he holds a bad project, then he rejects any contract. The price  $\overline{R}_0^i$ , which only informed entrepreneurs with good projects accept, is the highest price an entrepreneur will ever accept. The price  $\underline{R}_0^i$ , which is accepted by all entrepreneurs (except the informed entrepreneur with a bad project), is the highest price an uninformed entrepreneur will accept.

Our next result further simplifies our analysis as it shows that the investor's decision under uninformed financing is to choose among two possible contracts.

**Lemma 4** *In any PBE, the investor chooses  $R_0^U \in \{\underline{R}_0^U, \overline{R}_0^U\}$ .*

Our setup makes it easy to constrain the investor's pricing strategy to a space in which only two numbers have positive probability mass, making the problem more tractable. The investor does not need to worry about financing an informed entrepreneur with a bad project.

However, if the investor requires a low share of the cash flow as payment, he cannot avoid financing uninformed entrepreneurs with bad projects. On the other hand, if he charges a high price, he will lose good deals from entrepreneurs that are unaware of the quality of their projects, those of type  $\{U\}$ .

**Lemma 5** *There is no VC-equilibrium in which all VCs report the same information.*

This result is also fairly intuitive. If all VCs reported the same information, by consistency of beliefs, the investor would consider the true probability distribution. In this case, the investor would be better off not spending any positive amount on screening services.

The next lemma will be important in deriving the incentives for the VC to report his information accurately. It implies that the investor has enough money to finance the entrepreneur's project in the second period if and only if the investment in period 0 succeeds.

**Lemma 6** *Suppose  $\pi_H - \frac{m_H + \bar{u}}{\lambda p} > 1$ . Then in any PBE the condition  $1 \leq (d - (1 - m_H) + R_0^i)$  for  $i = U, I$  is satisfied.*

We can now investigate the investor's problem under uninformed financing. Consider  $P(R_0^U)$  the investor's expected repayment given that he chooses a contract with price  $R_0^U$ . Let  $Q(R_0^U)$  be the probability that an entrepreneur with a good project accepts a contract with price  $R_0^U$ . We can think of  $P$  as the "price" and  $Q$  as the "quantity" in a standard demand framework. Figure 1.3 shows that the investor in our model faces a standard monopoly dilemma. The bold vertical lines represent the entrepreneur's demand. The investor would like to discriminate and charge a high price from an entrepreneur of type  $\{G\}$  and a low price from an entrepreneur of type  $\{U\}$ . However, since types are unobservable, the investor needs to set a flat price. If the price chosen is  $\bar{R}_0^U$ , then the investor's expected revenue is given by

the area  $A+B+C$ , while his expected cost is given by  $C$ . If the price is  $\underline{R}_0^U$ , then the investor's expected revenue and cost are given by the areas  $B+C+D+E$  and  $C+E+F$ , respectively.

[Figure 1.3 about here]

Under uninformed financing, the investor's expected payoff when he chooses  $R_0^U = \overline{R}_0^U$  is given by  $(A+B+C) - C$ :

$$\overline{\Pi} = \lambda q \left[ p \left( \pi_H - \frac{m_H + \bar{u}}{p} \right) + p(p\pi_H - \bar{u} - 1) \right] - \lambda q(1 - m_H). \quad (1.6)$$

On the other hand, if he chooses  $\underline{R}_0^U$ , the ex ante expected payoff is  $(B+C+D+E) - (C+E+F)$ :

$$\underline{\Pi} = \lambda \left[ p \left( \pi_H - \frac{m_H + \bar{u}}{\lambda p} \right) + p(p\pi_H - \bar{u} - 1) \right] - (\lambda q + (1 - q))(1 - m_H). \quad (1.7)$$

Note that the investor would always choose  $\overline{R}_0^U$  if  $q \rightarrow 1$  or  $\lambda \rightarrow 0$ . If the entrepreneur is informed, he will accept a contract if and only if he has a good project. Hence, it is a weakly dominant strategy to charge  $\overline{R}_0^U$ . When the entrepreneur has a bad project, the investor can guarantee himself a revenue of zero by choosing  $\overline{R}_0^U$ . If he chooses  $\underline{R}_0^U$ , his expected revenue will be negative. On the other hand, the investor would always choose  $\underline{R}_0^U$  if  $\lambda \rightarrow 1$ . In this case, we would have  $0 < \overline{\Pi} = q\underline{\Pi} < \underline{\Pi}$ . Also notice that the higher the values of  $p$ ,  $\lambda$ , and  $\pi_H$ , the more likely it is for the investor to choose  $\underline{R}_0^U$ . Intuitively, the investor's choice of whether to send a contract to the VC is determined by a trade-off between the price that he has to pay and his belief about the accuracy of the VC's information.

Once the outcome under uninformed financing (the investor's outside option) is determined, we need to compare it with the outcome he would get under informed financing.

Under informed financing, the investor's expected payoff gross of  $z$  is:

$$\begin{aligned}
\Pi = & \theta \lambda q \left[ p R_0^I(\tau_{\{G\}}) + p(p\pi_H - \bar{u} - 1) \right] + \\
& (1 - \gamma)(1 - q) \lambda \left[ p R_0^I(\tau_{\{U\}}) + p(p\pi_H - \bar{u} - 1) \right] + \\
& (1 - \theta) \lambda q e_{\{U\}}^I(R_0^I(\tau_{\{G\}})) \left[ p R_0^I(\tau_{\{G\}}) - (1 - m_H) + p(p\pi_H - \bar{u} - 1) \right] + \\
& \gamma(1 - q) e_{\{U\}}^I(R_0^I(\tau_{\{U\}})) \left[ \lambda p R_0^I(\tau_{\{U\}}) - (1 - m_H) + \lambda p(p\pi_H - \bar{u} - 1) \right] - \\
& (1 - \theta)(1 - \lambda) q e_{\{U\}}^I(R_0^I(\tau_{\{B\}})) (1 - m_H) - \lambda q(1 - m_H).
\end{aligned} \tag{1.8}$$

The intuition for our results is captured by Figure 1.3. First, suppose the types of the entrepreneur and the VC are perfectly correlated; i.e.,  $(\theta, \gamma) = (1, 1)$ . Second, assume  $A - (D - F) > 0$ ; i.e.,  $\bar{\Pi} > \underline{\Pi}$ . Third, consider a separating equilibrium  $\tau_{\{B\}} \neq \tau_{\{U\}} \neq \tau_{\{G\}}$ . In this economy, the investor will choose  $\bar{R}_0^U$  if he finances the entrepreneur directly, receiving an expected payoff of  $\bar{\Pi}$ . However, if he uses VCs as intermediaries and chooses  $R_0^I(\tau_{\{G\}}) = \bar{R}_0^U$  and  $R_0^I(\tau_{\{U\}}) = \underline{R}_0^U$ , his expected payoff (gross of  $z$ ) will be  $A + B + D - F$ :

$$\Pi = \bar{\Pi} + (1 - q) \bar{\Pi} (q = 0) = \underline{\Pi} + q(1 - \lambda)(m_H + \bar{u}). \tag{1.9}$$

The investor will find it attractive to hire the VC whenever  $D - F$  greater than zero. The difference is the surplus generated by screening. When the surplus is smaller than the amount of resources left to the investor after financing, the VC will charge the investor the entire surplus, which is denoted by  $z$  in the figure. Otherwise he will charge  $d - (1 - m_H)$ .

We can think of two concepts when analyzing the efficiency of an outcome. The first concept is that of ex post efficiency, in which case we check for efficiency given that the quality

of the entrepreneur’s project is known. Here, ex post efficiency requires good projects being financed with probability one and bad projects with probability zero. The second concept is that of ex ante efficiency; i.e., efficiency is checked before the quality of the entrepreneur’s project is known. Ex ante efficiency requires uninformed entrepreneurs to be financed if  $D - F > 0$ .

In bad times (low  $\lambda$ ), we are more likely to observe  $\bar{\Pi} > \underline{\Pi}$ . It follows that uninformed financing is ex post inefficient as good projects will be financed with probability  $q$ . If  $D - F > 0$ , then uninformed financing is also ex ante inefficient as uninformed entrepreneurs are not financed. In good times (high  $\lambda$ ), we are more likely to observe  $\bar{\Pi} < \underline{\Pi}$ . Bad projects will be financed with probability  $1 - q$ , which implies that uninformed financing is ex post inefficient. However, in this case uninformed financing is ex ante efficient if  $D - F > 0$ .

Another interesting implication of our results is that when the market conditions are favorable — large fraction of good projects, high chance of success, and large payoffs — the investor will demand a lower share of the gains from entrepreneurs. The reason is simple: when market conditions are attractive and the investor charges a high price, he is heavily penalized by not financing uninformed entrepreneurs with good ventures. This cost is reduced when market conditions deteriorate and the probability of financing a uninformed entrepreneur with bad project increases.

Our objective now is to determine when there is a surplus from screening. This depends on the accuracy of the report produced by the VC. The incentive of the VC to report accurately depends on his information and on the future gains from the partnership. We first examine the case in which the VC has “industry and entrepreneur-type expertise.” In this

case, the types of the entrepreneur and the VC are perfectly correlated. This simpler setup serves as the baseline model since the VC has incentive to report truthfully and the outcome will be that of standard price discrimination. We then study the case in which VCs have only “industry expertise.” In this more realistic setting, the VC might misreport in equilibrium in order to maximize his carry. Interestingly, the information might be accurate enough so as to adjust the beliefs of the investor and the entrepreneur, in which case informed financing generates a surplus to the investor.

### 1.3.2 Industry Expertise and Entrepreneur-Type Expertise

This represents the case where the VC can be informed about the project’s quality and about the entrepreneur’s knowledge about the project’s quality. In this subsection, we characterize the VC-equilibria. The expected surplus from screening will be calculated and the existence of a VC-equilibria will be established. A necessary and sufficient condition for emergence of VCFs is that the expected surplus from financing an uninformed entrepreneur is positive. This condition is readily available from Figure 1.3 and is equivalent to  $D - F > 0$ .

We characterize VC-equilibria by their types and outcomes. The types of VC-equilibria are: *separating* equilibria, where  $|S(\tau)| = 1 \forall \tau \in T$ ; and *semi-pooling* equilibria, where  $|S(\tau)| = 2$  for some  $\tau \in T$ . With slight abuse of notation, we denote  $\tau$  as the report given by the two types of VC that pool in a semi-pooling equilibrium, and  $\tau_{\iota_V}$  as the report of the separating type  $\iota_V$ . Without loss of generality, we will consider the cases  $\bar{\Pi} > \underline{\Pi}$ , with the set of VC-equilibria denoted by  $A(\bar{\Pi})$ , and  $\underline{\Pi} > \bar{\Pi}$ , with the set of VC-equilibria denoted by  $A(\underline{\Pi})$ . The outcome of a VC-equilibrium is determined by the investor’s action played in

equilibrium as a function of VC types  $R_0^I(\tau_{lv})$  and by the price  $z$  charged by VCs.

We now establish the first major result of our paper, which is the characterization of VC-equilibria when VCs have both industry and entrepreneur-type expertise.

**Proposition 1** *A VC-equilibrium exists if and only if  $\underline{\Pi}(q=0) > 0$ . If a VC-equilibrium exists, then each of the sets  $A(\bar{\Pi})$  and  $A(\underline{\Pi})$  contains three types of equilibria with a unique outcome: separating equilibria; semi-pooling equilibria with  $S(\tau) = \{\{U\}, \{B\}\}$ ; and semi-pooling equilibria with  $S(\tau) = \{\{G\}, \{B\}\}$ . The unique outcome is given by:*

$$z = \begin{cases} \min \{d - (1 - m_H), (1 - q) \underline{\Pi}(q=0)\} & \text{if } \bar{\Pi} > \underline{\Pi} \\ \min \{d - (1 - m_H), q(1 - \lambda)(m_H + \bar{u})\} & \text{if } \underline{\Pi} > \bar{\Pi} \end{cases},$$

$$R_0^I(\tau_{\{G\}}) = \bar{R}_0^I,$$

$$R_0^I(\tau_{\{U\}}) = \underline{R}_0^I(\tau_{\{U\}}) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}.$$

The assumption  $\underline{\Pi}(q=0) > 0$  means that the expected surplus upon financing an entrepreneur who is known to be uninformed about his project is positive. An interesting insight from this proposition is that a truth-telling equilibrium is possible where the investor associates with a VC to form a VCF and VC reports accurately reflect the quality of the projects being financed.

**Corollary 1** *If  $\bar{\Pi} > \underline{\Pi}$ , then  $z$  is non-decreasing in  $p, \lambda, \pi_H, m_H, d$ , and it is non-increasing in  $q$  and  $\bar{u}$ .*

Intuitively, if the investor is charging  $\bar{R}_0^U$ , then the information becomes more valuable as the probability of good projects increases. This happens because it becomes more likely that uninformed types with good projects will reject a price  $\bar{R}_0^U$ . Thus, knowing the true state makes it possible to increase profits. In addition, as the probability of success increases, the loss from not financing an uninformed entrepreneur with a good project also increases, making the information more valuable. The same effect explains the relationship between  $z$  and

the outcome of good projects. On the flip side, when  $q$  increases, the possibility of mistakes — i.e., not financing a good project — becomes smaller, making the information less valuable.

**Corollary 2** *If  $\bar{\Pi} < \underline{\Pi}$ , then  $z$  is increasing in  $m_H$ , non-decreasing in  $q$ ,  $\bar{u}$ ,  $d$ , and non-increasing in  $\lambda$ .*

If the investor charges  $\underline{R}_0^U$ , the information becomes more valuable as the probability of being informed increases. This happens because it becomes less likely that an uninformed entrepreneur with a good project will not be financed. At the same time, the potential gains associated with rents extracted from informed entrepreneurs with good projects increase. Therefore, charging  $\bar{R}_0$  increases profits. On the other hand, when  $\lambda$  increases, charging  $\underline{R}_0^U$  reduces the possibility of mistakes (not financing a good project), making the information less valuable.

It is worth noting that these results have interesting welfare implications. Recall that if  $\bar{\Pi} > \underline{\Pi}$ , then (1) uninformed financing is ex post inefficient since good projects are financed with probability  $q$ , and (2) uninformed financing is ex ante inefficient if  $D - F > 0$  as uninformed entrepreneurs are not financed. Although informed financing is still ex post inefficient (bad projects are financed with probability  $1 - q$ ), ex ante efficiency is increased since uninformed entrepreneurs are now financed in equilibrium. Therefore, although the VC may not necessarily have a better understanding of the venture (relative to the entrepreneur), they help reduce financing inefficiencies.

If  $\bar{\Pi} < \underline{\Pi}$ , then (1) uninformed financing is ex post inefficient since bad projects are financed with probability  $1 - q$ , and (2) uninformed financing is ex ante efficient if  $D - F > 0$  as uninformed entrepreneurs are financed. Therefore, there is no efficiency gain under in-



formed financing. However, there are important redistributive effects as the surplus from the entrepreneur of type  $\{G\}$  is transferred to the VC.

### 1.3.3 Industry Expertise

We now demonstrate that a VC-equilibrium may emerge even when the VC has more limited information about how knowledgeable the entrepreneur is about the quality of the project. The difference between this more realistic setting and that of the last subsection is that the VC is no longer fully informed. To be precise, the VC might know about the business of the entrepreneur, but he does not know whether the entrepreneur is aware of the quality of his project. This setting captures the situation where the VC is himself a former entrepreneur (“knows the business”), but he has no special expertise in assessing the knowledge of entrepreneurs currently seeking finance. A direct consequence of this information structure is that a truth-telling VC-equilibrium along with a price discrimination outcome will no longer obtain. However, an interesting mechanism comes into play. The idea is illustrated in Figure 1.4.

[Figure 1.4 about here]

Figure 1.4 contains the same investor’s dilemma under uninformed financing that is featured in Figure 1.3. Because a truth-telling VC-equilibrium is unlikely, the investor can no longer rely on the reports of the VC to identify the entrepreneurs of type  $\{G\}$  and  $\{U\}$ . However, the reports from the VC might be accurate enough so as to adjust the expectations of the entrepreneur and the investor. For instance, suppose the VC reports  $\{G\}$  whenever his type is either  $\{G\}$  or  $\{U\}$ , and reports  $\{B\}$  whenever his type is  $\{B\}$ . First, consider the case in

which the entrepreneur holds a good project but is unaware of that; i.e., he is of type  $\{U\}$ . In this case, upon receiving a report from the VC saying that the entrepreneur's project is good, the uninformed entrepreneur believes with higher probability that he holds a good project. As a consequence, the uninformed entrepreneur would be willing to accept a contract with repayment  $\underline{R}_0^I$ , which is higher than the possible repayment  $\underline{R}_0^U$  under uninformed financing. The investor also updates his beliefs and will charge  $\underline{R}_0^I$  under informed financing. Therefore, the investor's revenue would be  $G + H$  higher compared to that under uninformed financing with repayment  $\underline{R}_0^U$ . Second, consider the case in which the entrepreneur holds a bad project but is of type  $\{U\}$ . Upon receiving a report from the VC saying that the entrepreneur's project is bad, both the uninformed entrepreneur and the investor know the project is bad. As a result, the project will not be financed and the expected financing cost  $I$  is subtracted from the total expected cost under uninformed financing with repayment  $\underline{R}_0^U$ .

Under the scenario described above, it is straightforward to conclude that, if  $\bar{\Pi} < \underline{\Pi}$ , then a VC-equilibrium exists. In addition, if  $\bar{\Pi} > \underline{\Pi}$  instead, then a VC-equilibrium exists if and only if  $G + H + D - (F - I) - A > 0$ . We now establish the second main result of our model, which is the characterization of equilibria when VCs have only industry expertise.

**Proposition 2** (i) *If  $\bar{\Pi} > \underline{\Pi}$ , then a VC-equilibrium exists. The set  $A(\bar{\Pi})$  contains two types of equilibria with a unique outcome: separating equilibria; and semi-pooling equilibria with  $S(\tau) = \{\{U\}, \{B\}\}$ . The unique outcome is given by:*

$$\begin{aligned} z &= \min \{d - (1 - m_H), (1 - q) \bar{\Pi}\}, \\ R_0^I(\tau_{\{G\}}) &= \underline{R}_0^I(\tau_{\{G\}}) = \bar{R}_0^I, \\ R_1(\tau_{\{U\}}) &= \bar{R}_0^I. \end{aligned}$$

(ii) *The set  $A(\bar{\Pi})$  contains semi-pooling equilibria in which  $S(\tau) = \{\{G\}, \{U\}\}$  if and*

only if

$$\underline{\Pi}(q=0) - \frac{[q(1-\lambda)]^2}{1-q(1-\lambda)}(m_H + \bar{u}) \geq 0.$$

The resulting outcome is given by:

$$\begin{aligned} z &= \min \{d - (1 - m_H), (1 - q) \underline{\Pi}\}, \\ R_0^I(\tau) &= \underline{R}_0^I(\tau) = \pi_H - [1 - q(1 - \lambda)] \frac{m_H + \bar{u}}{\lambda p}. \end{aligned}$$

(iii) *Semi-pooling equilibria in which  $S(\tau) = \{\{G\}, \{B\}\}$  do not exist.*

As the proposition states, we have the possibility of two equilibria with different outcomes. We use the refinement proposed by Matthews et al. (1991) for cheap-talk games to show that the outcome described in (ii) of Proposition 2 is the most sensible one. We formally discuss and define the refinement criterion in Appendix B.

The basic equilibrium argument is as follows. Suppose  $\bar{\Pi} > \underline{\Pi}$  such that the investor would charge  $\bar{R}_0^U$  under uninformed financing. In this case, uninformed entrepreneurs with good projects would reject contracts. Consider a truth-telling VC-equilibrium. In this separating equilibrium, uninformed entrepreneurs with good projects learn about the quality of their projects. This allows investors to demand  $\bar{R}_0^I$  from these entrepreneurs and increase their expected payoffs. Since the signal of the VC of type  $\{U\}$  is uninformative, the investor also charges  $\bar{R}_0^I$  upon receiving a report from a VC of type  $\{U\}$  (since this was optimal under uninformed financing).

Now consider a VC-equilibrium in which the VC of type  $\{U\}$  pools with the VC of type  $\{G\}$ . Under this equilibrium, it must be the case that the investor charges  $\underline{R}_0^I(\tau)$  upon receiving report  $\tau$  from either the VC of type  $\{G\}$  or the VC of type  $\{U\}$ . This follows from the observation that, if the investor charged  $\bar{R}_0^I$ , only entrepreneurs of type  $\{G\}$  would

accept the contract, giving the investor the same expected payoff as under uninformed financing. The difference from the separating equilibrium is that, while the VC of type  $\{G\}$  has the same expected payoff, the VC of type  $\{U\}$  receives a higher expected carried interest. Therefore, one should expect the VC of type  $\{U\}$  to try to convince the investor that his type is in  $\{\{G\}, \{U\}\}$ . If he announces that his type is in  $\{\{G\}, \{U\}\}$ , the investor has no reason not to believe it since the equilibrium strategy of the VC of type  $\{B\}$  gives him a payoff that is at least as good as the best payoff he receives if the announcement is believed (he receives 0 in both cases). At the same time, the VC of type  $\{G\}$  is indifferent and could well be the author of the announcement. Therefore, the announcement  $\{\{G\}, \{U\}\}$  is credible. An equilibrium for which there is no credible announcement is called *strongly announcement-proof* (Matthews et al. (1991)).

**Proposition 3** *The equilibria in (i) of Proposition 2 are not strongly announcement-proof. The equilibria in (ii) of Proposition 2 are strongly announcement-proof.*

From a practical perspective, this is perhaps the main result of our model. It allows us to derive a number of comparative statics regarding the management fee charged by VCs. The model has very clear predictions about the relation between management fees and private equity activity. Investments in the VC industry are positively correlated with the fixed compensation received by VCs, which makes management fees pro-cyclical.

**Corollary 3** *For equilibria in (ii) of Proposition 2,  $z$  is increasing in  $m_H$ , non-decreasing in  $p$ ,  $\lambda$ ,  $\pi_H$ ,  $d$ , and non-increasing in  $\bar{u}$ . Moreover, there exists a  $\lambda^* \in (0, 1)$  such that  $z$  non-increasing in  $q$  for  $\lambda > \lambda^*$  and non-decreasing in  $q$  for  $\lambda < \lambda^*$ .*

The main difference between this result and the one obtained in the previous subsection relates to  $q$ . When  $q$  increases, report  $\tau$  is more informative since it is more likely to come

from the VC of type  $\{G\}$  than from the VC of type  $\{U\}$ . Uninformed entrepreneurs would believe with higher probability that they hold a good project, which would allow the investor to charge a higher price. However, when  $\lambda$  is reasonably high, increases in  $q$  greatly reduce the potential loss from not financing an uninformed entrepreneur with a good project. At the same time, for low levels of  $\lambda$ , increases in  $q$  not only make report  $\tau$  more informative, but also increase the potential loss from not financing uninformed entrepreneurs with good projects. Therefore, the information becomes more valuable.

We now establish the last key result of the model, characterizing equilibria when VCs have only industry expertise and  $\underline{\Pi} > \bar{\Pi}$ .

**Proposition 4** (i) *If  $\underline{\Pi} > \bar{\Pi}$ , then a VC-equilibrium exists. The set  $A(\underline{\Pi})$  contains two types of equilibria with a unique outcome: separating equilibria; and semi-pooling equilibria with  $S(\tau) = \{\{G\}, \{U\}\}$  if and only if*

$$\underline{\Pi}(q=0) - \frac{[q(1-\lambda)]^2}{1-q(1-\lambda)}(m_H + \bar{u}) \geq 0.$$

*The unique outcome is given by:*

$$\begin{aligned} z &= \min \{d - (1 - m_H), q(1 - \lambda)[(1 - q)(1 - m_H) + m_H + \bar{u}]\}, \\ R_0^I(\tau_{\{G\}}) &= \underline{R}_0^I(\tau_{\{G\}}) = \bar{R}_0^I, \\ R_0^I(\tau_{\{U\}}) &= \underline{R}_0^I(\tau_{\{U\}}) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}. \end{aligned}$$

(ii) *Semi-pooling equilibria in which  $S(\tau) = \{\{G\}, \{B\}\}$  and  $S(\tau) = \{\{U\}, \{B\}\}$  do not exist.*

**Corollary 4**  *$z$  increasing in  $m_H$ , non-decreasing in  $\bar{u}$ , and non-increasing in  $\lambda$ .*

In this more realistic setup, informed financing increases both ex ante and ex post efficiency. If  $\bar{\Pi} > \underline{\Pi}$ , then uninformed financing is ex ante inefficient if  $D - F > 0$  as uninformed entrepreneurs are not financed. Under a VC-equilibrium, all uninformed entrepreneurs with

good projects are financed, and the probability that an uninformed entrepreneur with a bad project is financed is  $(1 - q)^2$ . Therefore, informed financing increases ex ante efficiency.

If  $\bar{\Pi} < \underline{\Pi}$ , then uninformed financing, although ex ante efficient (if  $D - F > 0$ ), is ex post inefficient as a bad project is financed with probability  $1 - q$ . However, the probability that a bad project is financed under a VC-equilibrium is only  $(1 - q)^2$ . As a result, VCs increase both ex ante and ex post efficiency in the financial markets.

Notably, although VCs do not report truthfully when they are uninformed about the project, they reduce inefficiencies. What drives this result is the fact that the VC may know about the quality of the project even if the entrepreneur is uninformed. This result is interesting in highlighting how financing efficiency can be improved in equilibrium even when agents produce less than perfect reports.

## 1.4 Model Calibration

We calibrate our model for the case in which VCs have industry expertise. We focus on this case because it requires the least amount of knowledge by the VCs and yet it offers enough conditions to identify the model parameters. The industry *only* expertise scenario also strikes us as the more interesting one. Our numbers come largely from the study of Sahlman (1990) on the venture capital industry. An individual investment is characterized by the funding given to a single project throughout the financing cycle and by the length of the financing cycle. We take that the representative real-world counterpart of that project is the average start-up company financed by VCs. The average life of an investment in Sahlman is 5 years and the results derived here should be interpreted with that time horizon in mind.

In the model, the price charged by the VC is independent of the success of the venture that is financed. The same is true regarding the management fees that VCs charge in the real world. In addition, VCs usually get 20% of the profits in case of success. The counterpart in the model is the VC's reservation utility for carrying out the project. The VC receives that amount only if the investment does not fail in period 0.

Our first step is to calibrate  $\pi_H$ . Sahlman reports that around 16% of the investments made are responsible for 75% of the ending value of the portfolio. Moreover, about 34% of the ventures sponsored by the VCs fail. The remaining 50% of the investments account for 25% of the total ending value. He also finds that the ending value of investments is 4.3 times the original cost.

One can represent the numbers in Sahlman assuming that the ending value of the portfolio is as a random variable that takes values 0, 2.15, and 20.156 with probabilities 0.34, 0.50, and 0.16 respectively. We can represent this by points  $(x, y)$  in  $\mathbb{R}^2$  such that  $(x, y) \in \Sigma = \{(0.16, 20.156), (0.34, 0), (0.50, 2.15)\}$ . Let us define  $\Gamma = \sum_{\{x:(x,y) \in \Sigma\}} |f(x) - y|$ . In our model, an investment takes the values  $\pi_H$  and  $\pi_L$ . Accordingly, we want to calibrate  $\pi_H$  and  $\pi_L$  in order to concisely approximate the set  $\Sigma$ . One way to do this is by using a step function with two steps. We define an optimal step function as one that minimizes  $\Gamma$ .<sup>7</sup> An optimal two-step function approximation  $f : [0, 1] \rightarrow \mathbb{R}$  that minimizes  $\Gamma$  is given by:

$$f(x) = \begin{cases} 20.156, & \text{for } x \in [0, 0.34) \\ 0, & \text{for } x \in [0.34, 1] \end{cases} . \quad (1.10)$$

---

<sup>7</sup>Another way of fitting a step function to a point set is by minimizing the maximum vertical difference (see Fournier and Vigneron (2008)). Our conclusions are similar if we use this alternative approach.

We want an approximation such that the average ending value of investments is equal to 4.3. Accordingly, we need a two-step function  $f'$  that minimizes  $\Gamma$  subject to the constraint

$\sum_{\{x:(x,y) \in \Sigma\}} x f'(x) = 4.3$ . It is straightforward to see that the following function satisfies that requirement:

$$f'(x) = \begin{cases} 26.875, & \text{for } x \in [0, 0.34) \\ 0, & \text{for } x \in [0.34, 1] \end{cases} . \quad (1.11)$$

We can model the ending value of a portfolio of projects as a Bernoulli process that takes values  $\pi_H = 27$  in case of success, and  $\pi_L = 0$  in case of failure. This gives us the best two-step function approximation to the actual ending value distribution, subject to the constraint that the average implied by the model matches the data.

We use additional information from the VC industry. In particular, note that in 88% of the funds surveyed by Sahlman, VCs are entitled to 20% of the realized gains. Since the average expected net gain from an investment is 3.3 ( $= 4.3 - 1$ ), we set  $\bar{u} = 0.66$ . The average share of the ending portfolio held by the founders of the venture (entrepreneurs) is roughly 30% (see Sahlman (1990) and Kaplan and Strömberg (2003)). Therefore,  $m_H$  solves  $m_H + 0.66 = 0.3 \times 3.3$ , which implies  $m_H = 0.33$ .

Since 34% of the ventures sponsored by VCs are expected to fail, we have an estimate of the probability that an investment fails given that it is financed by VCs. The equilibrium of our model implies that a bad project will be financed with probability  $(1 - \lambda)(1 - q)^2$  and a good project will be financed with probability  $\lambda$ . Hence, the following condition must be



satisfied:

$$0.34 = \Pr(\text{failure} \mid \text{financed by VCFs}) = \frac{(1 - \lambda)(1 - q)^2 + \lambda(1 - p)}{(1 - \lambda)(1 - q)^2 + \lambda}. \quad (1.12)$$

From Berlin (1998), we take that the probability that a VC funds a received project is 10%. In our model, the probability that a project will be financed by the VCF is  $(1 - \lambda)(1 - q)^2 + \lambda$ . As a result, we have the following condition:

$$0.10 = \Pr(\text{funding}) = (1 - \lambda)(1 - q)^2 + \lambda. \quad (1.13)$$

Our next condition comes from the probability of failure given that projects are financed. One of the main distinctions between VC-financed and non-VC-financed firms is that the former typically have low cash flows and do not have tangible assets to offer as collateral. Puri and Zarutskie (2008) show that 47% of firms financed by VCs have zero cash revenue in their first year, compared to only 6% of firms with other sources of financing. They also report evidence that most of the difference between failure rates of VC-financed and non-VC-financed firms is due to successful selection of good projects by VCs. In other words, firms that seek VC financing are drawn from the general distribution of start-up firms in the economy, but VCs seem to add value by way of their selection process. Phillips and Kirchoff (1989), Puri and Zarutskie (2008), and Bernhardt and Krasa (2008) estimate that the failure rate of all start-up firms, firms financed outside the VC industry, and all firms that seek finance from VCs are, respectively, 60%, 62%, and 57%. Since firms that seek financing

from VCs are thought to be riskier than the average firm, we use the following condition:

$$0.62 = \Pr(\text{failure} \mid \text{all financed}) = \frac{(1 - \lambda)(1 - q) + \lambda(1 - q)(1 - p) + \lambda q(1 - p)}{(1 - \lambda)(1 - q) + \lambda(1 - q) + \lambda q}. \quad (1.14)$$

Using conditions (7), (8), and (9) we can solve for  $q$ ,  $\lambda$ , and  $p$ :

$$\lambda = 0.093, p = 0.711, q = 0.911. \quad (1.15)$$

These conditions imply an environment in which the probability of good projects is low. However, given that the project is of good quality, its probability of success is high. This result highlights the consistency of our model with the VC industry and provides insights into the economic role of VCs. In essence, VCs are agents that can screen out bad projects in an environment where good projects are scarce. They add value to investors and contribute to the capital allocation process.

Lastly, we normalize the investor's money endowment to one; i.e.,  $d = 1$ . We summarize the calibration of the parameters in Table 1.1.

[Table 1.1 about here]

We can now compute the endogenous variables of the model:

$$\overline{\Pi} = 2.537, \quad (1.16)$$

$$\underline{\Pi} = 1.834, \quad (1.17)$$

$$\overline{R}_0^I = \left( \pi_H - \frac{m_H + \bar{u}}{p} \right) = 25.608, \quad (1.18)$$

$$\underline{R}_0^I = \pi_H - [1 - q(1 - \lambda)] \frac{m_H + \bar{u}}{\lambda p} = 24.395. \quad (1.19)$$

These estimates suggest that the investor would have charged 25.61 if he did not have the option to buy information from the VC. In other words, if the project succeeded, the entrepreneur would have to give the investor 25.61 from the realized gain of 27. However, Propositions 2 and 3 say that the investor will buy the services from the VC at a price determined by:

$$\Pi = \bar{\Pi} + (1 - q) \underline{\Pi} = 2.701, \quad (1.20)$$

$$z^* = \min \{1 - (1 - m_H), \Pi - \bar{\Pi}\} = \min \{0.33, 0.164\} = 0.164. \quad (1.21)$$

We note that these estimates match the real-world data quite well. The committed capital in our model is given by  $d = 1$ , which is the amount of resources the investor is willing to commit to the VCF at time 0. Accordingly, the estimations imply that the management fee charged by VCs is equal to 16.4% of committed capital. This is consistent with the numbers estimated by Metrick and Yasuda (2010) for VCFs with annual management fees of 2% and 2.5%, which are respectively 16.1% and 20.2%. This estimate also falls well within the management fee range of 15–20% that is reported by Gompers and Lerner (1999). Our estimated return of a VCF (net of fees) over its life is 2.70. The implied effective annual rate of return, in a five-year horizon, is 22%.<sup>8</sup> As it turns out, this number is virtually the same as those reported by Sahlman (1990) and Kaplan and Schoar (2005).

Finally, the expected carried interest received by the VC is given by

$$\lambda p \bar{u} = 0.044, \quad (1.22)$$

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<sup>8</sup>The effective rate of return  $r$  is given by  $2.7 = (1 + r)^5$ .

which falls short of the 0.073–0.083 range estimated by Metrick and Yasuda (2010) for VCFs with carry level of 20%. This underestimation is related to the interesting tension that exists in our model between management fees and carried interest. From Corollary 3, we know that the management fee is non-increasing in  $\bar{u}$ . However, the expected carried interest is increasing in  $\bar{u}$ . Therefore, an expected carried interest of 7% is associated with an estimated present value of management fees of 12.9%. This number is consistent with that estimated by Metrick and Yasuda for a management fee of 1.5%, which is 12.1%.

In summary, our model calibration matches key elements of the VC industry, such as expected returns and management fees. We estimate management fees to be 16.4% of committed capital and an annual rate of return of 22% for the industry. Notably, our model is consistent with the fact that the industry deals with risky projects. Only few projects have positive net present value, which makes the information and experience of VCs all the more valuable to investors. We believe the ability to represent these well-known stylized facts makes the model proposed particularly interesting.

## 1.5 Empirical Implications

Our calibration exercise allows us to sharpen the predictions of the model. In particular, it suggests that we may use Corollary 3 to derive implications regarding VC compensation. A non-exhaustive list of testable empirical implications derived from our model is as follows:

*Implication #1: Management fees should be higher for VCs that focus on early-stage and high technology ventures ( $z$  is increasing in  $m_H$ ).* The implication agrees with the notion that early-stage and high-technology ventures require higher entrepreneurial capital.

*Implication #2: Management fees should be higher for VCs specialized in industries in which good projects are more likely to succeed ( $z$  is non-decreasing in  $p$ ).* To the extent that VCs screen out bad projects from a pool of projects, these projects that are VC-financed are more likely to be good. Accordingly, if two identical VCs differ with respect to the industry in which they specialize, the one focused on the industry in which the return of VCFs first-order stochastically dominates that of the other should receive higher management fees.

*Implication #3: Management fees should be higher for VCs specialized in industries with higher incidence of good projects ( $z$  is non-decreasing in  $\lambda$ ).* VCs specialized in industries with smaller failure rates and higher returns should receive higher management fees.

*Implication #4: Management fees should be higher in booms and lower in busts ( $z$  is non-decreasing in  $\lambda$ ).* This result follows from the notion that booms (busts) are associated with higher (lower) incidence of good projects, which implies that management fees are procyclical.

*Implication #5: Management fees should be higher for VCs that manage funds in which successful investments generate higher payoffs ( $z$  is non-decreasing in  $\pi_H$ ).* If VCFs managed by two VCs share similar failure rates and differ from each other only regarding returns, then the VC associated with the funds with higher returns should receive higher management fees.

*Implication #6: Management fees should be higher for VCs that manage large funds ( $z$  is non-decreasing in  $d$ ).* VCs that manage VCFs with larger amounts of committed capital should receive higher management fees.

*Implication #7: The more experienced and skilled the VC, the lower the management fee*

*and the higher the carried interest.* This result exposes the tension that exists between the carry and the management fee. The tension arises because the opportunity costs of both the entrepreneur and the VC are linked through  $\bar{u}$ , which captures the characteristics of the investment and the industry where the investment takes place. The higher the  $\bar{u}$ , the higher the carried interest received by VCs. However, Corollary 3 shows that a higher  $\bar{u}$  also decreases the surplus brought about by VCs' screening, which in turn reduces the base compensation  $z$ .

Our predictions are consistent with a number of regularities reported in the empirical literature. The results in Gompers and Lerner (1999), for example, support Implications 1 and 7. VC organizations that focus on early-stage and high-technology investments tend to charge higher management fees. In addition, the management fee (carried interest) is negatively (positively) associated with size and age of VC organizations. Kaplan and Schoar (2005) find that industry returns (net of fees) are procyclical. This is in accordance with Implication 4 since management fees are given by the surplus brought about by VCs.

To our knowledge, many of the predictions listed above have not been empirically tested. This is somewhat surprising since they agree with heuristic arguments found elsewhere in the academic and practitioner literatures. Directly testing these model implications would deepen our understanding of the venture capital industry.

## 1.6 Concluding Remarks

Our paper analyzes the emergence and efficiency of venture capital partnerships. We develop a model in which a uniformed investor offers project funding and decides whether to screen an entrepreneur before financing. Financing takes place in two periods. Investment in the

first period requires resources from both the investor and the entrepreneur. Entrepreneurs are heterogeneous with respect to wealth, information, and project quality. Venture capitalists provide information for screening activity at a price. The relation that is created when the investor uses the services (i.e., buys information) from the VC gives rise to the venture capital fund (VCF).

We formally derive equilibrium conditions in which the investor pays VCs in order to carry out screening (*VC-equilibrium*). The willingness of the investor to buy information from VCs will depend on his expected return when he does not buy information and on his beliefs about the accuracy of the screening. The precision of screening will depend on the information of VCs and on the expected gains upon forming a VCF.

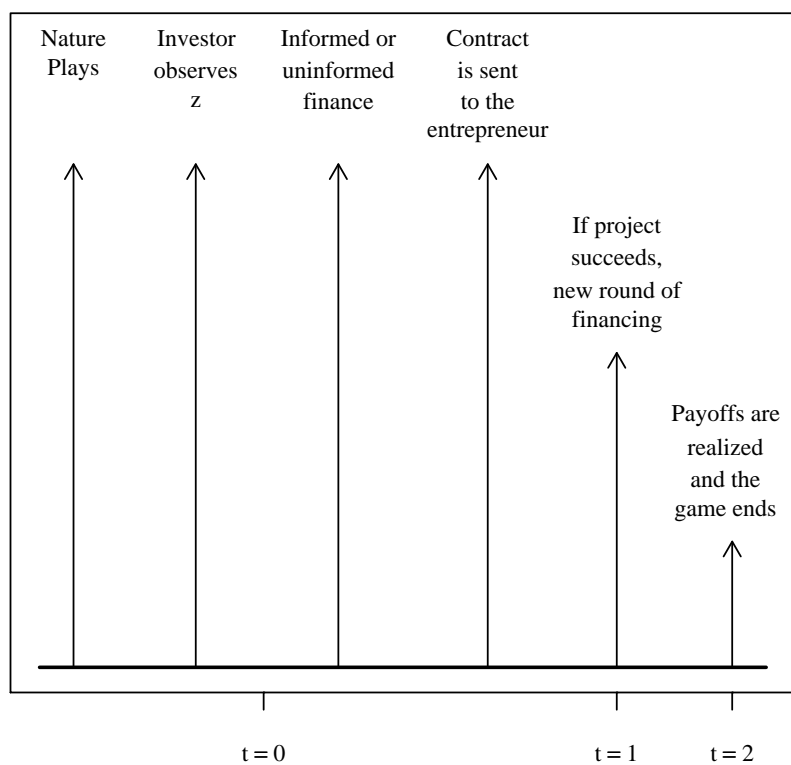
The analysis shows that uninformed financing allows for inefficient outcomes as good projects are not financed in bad times and bad projects are financed in good times. Although VCs might provide inaccurate reports to investors about investments opportunities, reports are precise enough so as to adjust beliefs of investors and entrepreneurs about projects. The correlation between the information of entrepreneurs and VCs is key to this adjustment. A lower proportion of bad projects is financed in good times and a higher proportion of good projects are financed in bad times — inefficiency is reduced.

In the final part of our analysis, we build on findings from previous studies to calibrate our model. We find estimates for the management fees charged by VCs and industry returns that are quite consistent with empirical evidence. We also provide a long list of testable model implications. To the best of our knowledge, most of those predictions remain untested. This is surprising since they agree with arguments found in the academic and practitioner literatures.

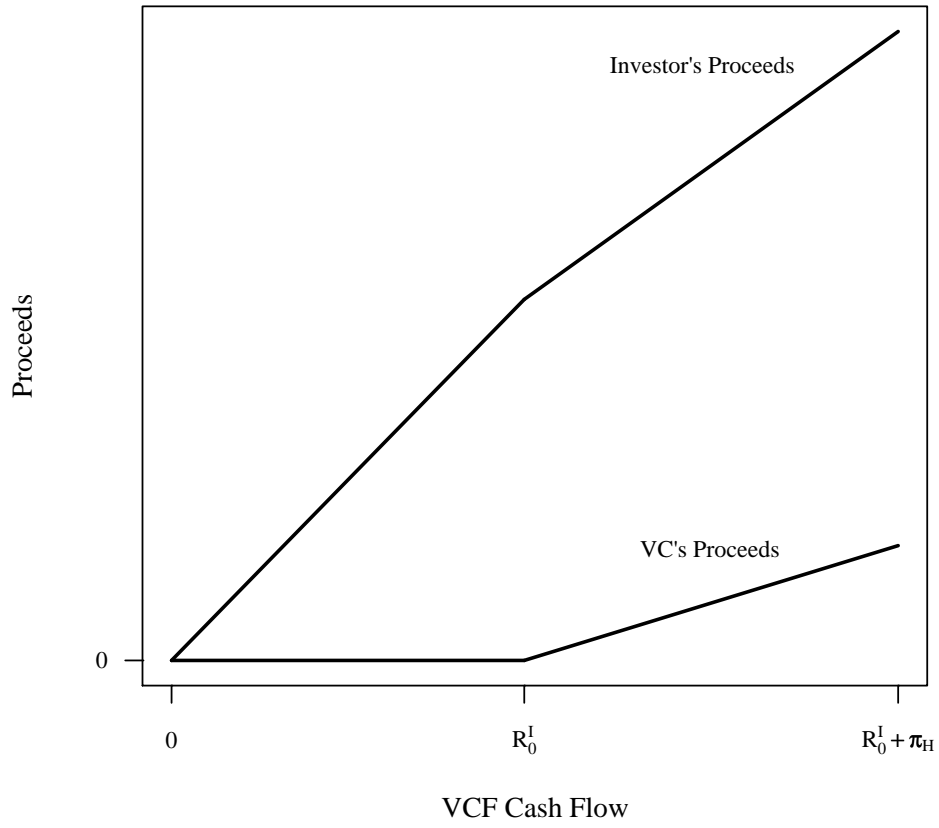
While there is much work to do on the economics of venture capital financing, the analysis of this paper reconcile a number of features of the VC industry within a standard contract theory framework. It also helps pushing theoretical and empirical research in the field in new directions. We believe analyses such as ours can be helpful in guiding regulatory and popular debates on the role of venture capital financing in the economy.



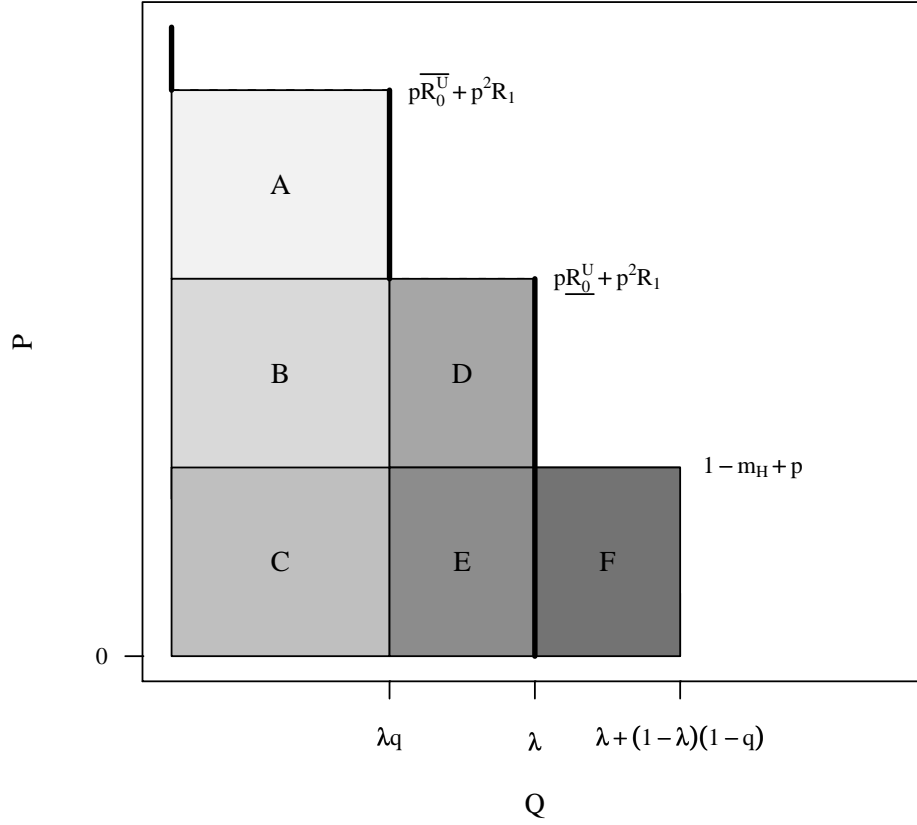
## 1.7 Figures and Tables



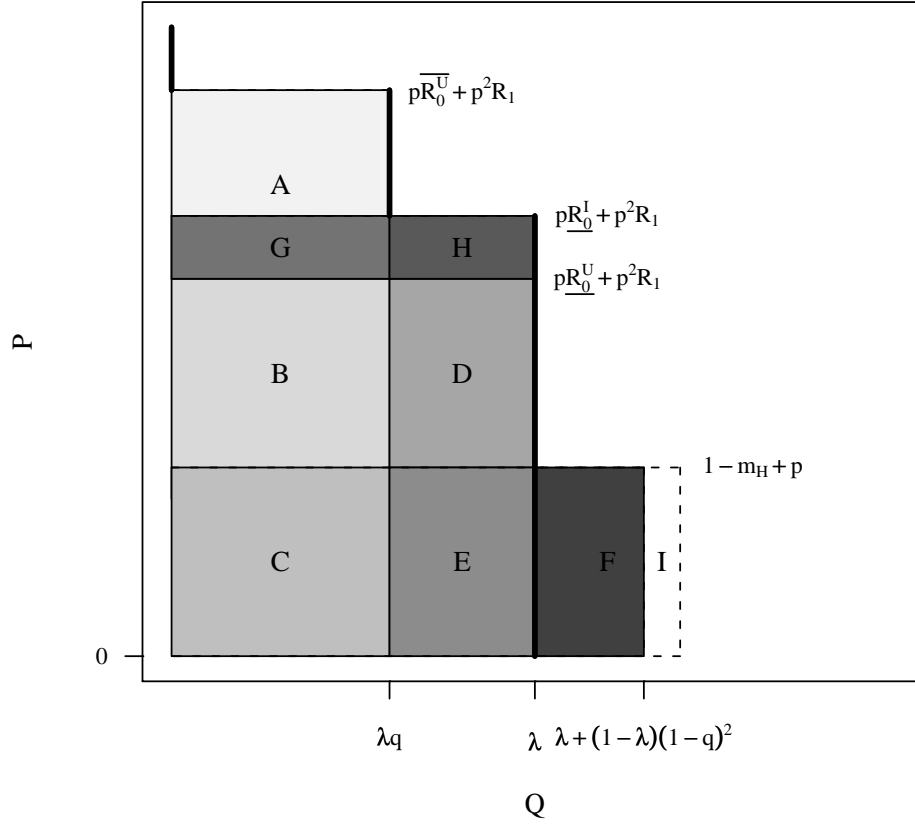
**Figure 1.1:** Timing of the Game



**Figure 1.2:** Investor's and VC's Proceeds. The first cash flow of the VCF is the investor's revenue from the investment in period 0, which is given by  $R_0^I$  and is appropriated by the investor. The second cash flow comes from the outcome of the second round of financing, which is given by  $\pi_H$  and is shared between the VC and the investor.



**Figure 1.3:** Outcomes under Uninformed Financing. The vertical axis represents the “price,” i.e., the investor’s expected repayment  $P(R_0^U)$  given his price strategy  $R_0^U$ . The horizontal axis represents the “quantity,” i.e., the probability that an entrepreneur with a good project accepts the contract  $Q(R_0^U)$ . The bold vertical lines represent the quantity demanded.  $R_1$  is the price of funding in the follow up stage of financing,  $p$  is the probability that a good project succeeds,  $q$  is the probability of being informed about the entrepreneur’s project,  $\lambda$  is the probability that the entrepreneur’s project has positive NPV, and  $m_H$  is the entrepreneur’s entrepreneurial capital.



**Figure 1.4:** Outcomes under Informed Financing. The vertical axis represents the “price,” i.e., the investor’s expected repayment  $P(R_0^i)$  given his price strategy  $R_0^i$  for  $i = U, I$ . The horizontal axis represents the “quantity,” i.e., the probability that an entrepreneur with a good project accepts the contract  $Q(R_0^i)$ . The bold vertical lines represent the quantity demanded at a given price.  $R_1$  is the price of funding in the follow up stage of financing,  $p$  is the probability that a good project succeeds,  $q$  is the probability of being informed about the entrepreneur’s project,  $\lambda$  is the probability that the entrepreneur’s project has positive NPV, and  $m_H$  is the entrepreneur’s entrepreneurial capital.

**Table 1.1:** Calibrated Parameters of the Model

$\pi_H$  is the return of a good project in the good state,  $q$  is the probability that the entrepreneur is informed,  $p$  is the probability that a good project succeeds,  $\lambda$  is the probability that the entrepreneur has a good project,  $\bar{u}$  is the entrepreneur's reservation utility,  $m_H$  is the high-type entrepreneurial capital, and  $d$  is the investor's amount of resources. See text for details of the parameter setting process.

$\pi_H$	$q$	$p$	$\lambda$	$\bar{u}$	$m_H$	$d$
27	0.911	0.711	0.093	0.66	0.33	1

# Chapter 2

## Credit Default Swaps, Firm Financing and the Economy

### 2.1 Introduction

The 2000s witnessed a formidable growth in the market for credit default swaps. According to the International Swaps and Derivatives Association (ISDA), the outstanding amount of CDS contracts grew from \$3 trillion in 2003 to a peak of \$62 trillion in 2007. The 2008–9 crisis brought attention to these contracts. There is an ongoing debate about whether CDSs contributed to the crisis and how one might regulate CDS markets.<sup>1</sup> Surprisingly, however, little is known about why CDSs exist in the first place. We know little about the role of CDSs in financial markets, what contracting inefficiencies they address, or whether they affect the availability of credit in the economy. Understanding these issues should strike anyone as an important step for improving financial architecture and regulation over the next decade.

A CDS is a bilateral agreement between a debt protection seller and a debt protection buyer. The buyer makes periodic payments to the seller in exchange for compensation in

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<sup>1</sup>Title VII of Dodd-Frank Act (HR #4173) gives the SEC regulatory authority over swaps, including CDSs. The Act requires the reporting of trades, sets position limits, imposes margin requirements, and moves swaps away from over-the-counter markets into organized exchanges.

the event a borrower defaults on its debt. Hu and Black (2008a,b) argue that CDSs can give rise to the *empty creditor problem*. Simply put, lenders protected by CDS might have low incentives to participate in out-of-court restructurings of distressed firms since formal default triggers immediate compensation for their exposure. The incentives to engage in restructuring could be even lower if lenders “overinsure,” that is, their protection payoff surpasses the amount of debt that can be recovered in default. In these cases, lenders might collect large profits from bankruptcy. CDS-insured lenders might thus force distressed firms into bankruptcy even when continuation would be optimal.<sup>2</sup>

The introduction of CDS contracts may alter the dynamics of corporate financing since optimal lending decisions are influenced by expected distress outcomes. While there is growing interest in the impact of CDSs on creditor–borrower relations, the literature lacks a model that examines important questions about these contracts. How do CDSs affect lenders’ preferences between out-of-court restructuring and bankruptcy? Do CDSs affect borrowers’ incentives to make their projects profitable and avoid bankruptcy? How do firm characteristics such as risk and size influence CDS contracting? How do economic conditions affect the demand for CDSs? Do CDS markets affect the availability of credit in the economy?

This paper develops a model of CDS contracting when investment is subject to moral hazard and verification is imperfect. To our knowledge, this is the first study to examine the optimal demand for CDS in a setting that incorporates these real-world complexities (we discuss the existing literature shortly). Creditors choose the amount of CDS protection to

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<sup>2</sup>Numerous accounts blame overinsured CDS lenders for blocking out-of-court restructurings of high profile firms during the financial crisis. In 2009 alone, companies in that category included Six Flags, Harrah’s, GM, Chrysler, Unisys, R. H. Donnelley, Abitibi Bowater, Marconi, and Lyondell Basell.

modulate their economic exposure to borrowers and our analysis shows how this choice is made, describing its economic consequences.

In a nutshell, our model shows that CDS overinsurance is associated with the implementation of efficient effort by borrowers, which maximizes the likelihood that projects succeed and alleviates the empty creditor problem. The model also shows that CDS overinsurance is more likely to be associated with safer firms. Along with evidence that CDSs are mainly written on these types of firms, our analysis suggests that CDS contracts may have emerged and become popular in the early 2000s precisely by virtue of its overinsurance capabilities. The analysis additionally implies that CDS contracts can boost the availability of credit in the economy and that CDS overinsurance is *less* common at times when projects are more likely to fail. Our paper shows that while CDSs facilitate borrowing by credit-constrained firms, CDSs will also be associated with their demise in bad times, leading to the “appearance” that CDSs aggravate the impact of economic downturns.<sup>3</sup> As we discuss below, proposed regulatory changes that prohibit lenders to overinsure via CDS may have the adverse consequence of reducing the availability of credit when firms most need it.

Let us provide context to our framework and discuss the implications of our analysis in some detail. In credit markets lenders have to design contracts that account for commitment issues, moral hazard problems, and inefficient restructuring protocols. Our analysis of CDS contracting incorporates these features. In the model, borrowers face a limited commitment problem in that they cannot commit to pay out cash flows from their projects. In effect, borrowers can divert cash flows and strategically trigger debt renegotiation even when projects

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<sup>3</sup>Relatedly, Stulz (2010) argues that it is unlikely that CDSs have caused the worst problems of the financial crisis.



succeed. Moreover, the amount of effort that borrowers dedicate to their projects is unobservable, even though that effort affects cash flows. Finally, we consider that restructuring is a costly process and can dissipate enterprise value.

As is standard, lenders can refuse to renegotiate contracts in default and force firms into liquidation; else they can engage in out-of-court restructurings and bargain over the portion of firm continuation values that can be verified. In the presence of CDS contracts, however, they have an alternative course of action. Lenders can insure against strategic renegotiation: CDS contracts trigger a payment by a third party if a “credit event” occurs.<sup>4</sup> As we demonstrate, the innovation brought about by CDSs is that they can be used to strengthen lenders’ position by: (1) increasing debt repayments when investments succeed and (2) increasing lenders’ share of proceeds in default states. Differently put, CDS insurance can be used to modulate whether lenders will have a stronger bargaining position when projects succeed or when they fail. Let’s discuss how this works under different degrees of CDS insurance.

If lenders buy CDS protection *beyond* the maximum amount they can receive in restructurings (i.e., lenders “overinsure” or have so-called “negative net economic ownership”), they pre-commit to forcing defaulting borrowers into bankruptcy. Intuitively, the mechanism works somewhat similarly to standard insurance. CDSs resemble actuarially fairly-priced policies and overinsurance increases both the likelihood that the insured party will require payoffs (immediate compensation for credit events) and the associated insurance premia (CDS fees).<sup>5</sup> To wit, once a credit event happens, the one-time payoff from seeking im-

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<sup>4</sup>As defined by the ISDA, credit events include default, debt acceleration, failure to pay, repudiation/moratorium, and bankruptcy. The standard CDS contract does not recognize out-of-court restructuring as a credit event.

<sup>5</sup>As in any competitive market, the insurance premium schedule is such that, in expectation, the insured

mediate borrower liquidation is large enough to commit CDS-protected lenders with that course of action. As a result, borrowers are prevented from capturing rents from default–continuation strategies. This not only discourages borrowers from defaulting strategically, but also incentivizes them to exert high effort, increasing the likelihood that investments are successful. By altering the dynamics of renegotiations and heightening borrowers’ incentives, overinsured lenders maximize regular debt repayments in good investment states (e.g., extraction of higher “debt coupons”).

If lenders buy an amount of CDS protection that *equals* the maximum payoff under restructuring (“zero net economic ownership”), they do not commit to unconditional liquidation in default states.<sup>6</sup> Instead, they position themselves so as to bargain over surpluses stemming from out-of-court renegotiations. Although zero net economic ownership maximizes the amount of debt repayment consistent with no liquidation, it leaves some surplus for borrowers when the verification of funds in default states is imperfect. Because “just-insured” lenders are relatively less inclined to call for bankruptcy if borrowers default, they pay lower fees for their CDS insurance. At the same time, because borrowers retain a fraction of restructuring values and know forced liquidation is less likely to happen, they are more prone to strategically default. This dynamic determines the tradeoffs faced by just-insured lenders. These lenders forego debt repayment surpluses that are extracted when investments succeed in exchange for higher renegotiation proceeds when investments fail.

The optimal degree of CDS insurance will be a function of tradeoffs between continuous party does not make an economic profit (e.g., higher insurance payoffs are associated with higher premia).

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<sup>6</sup>For completeness, in the law and economics literature “positive net economic ownership” refers to the case in which lenders do not completely hedge their economic exposure to borrowers (see Hu and Black (2007)). We will later discuss current proposals mandating that lenders maintain such positions in CDS markets.

tion and liquidation values, as well as the probability of investment success. When values under out-of-court restructuring and liquidation are similar, lenders expect to get the same payoff should firms become distressed. Given a similar bad state payoff, it is not worth it for lenders to position themselves so as to bargain over firm continuation. Instead, lenders will be inclined to overinsure so as to maximize gains from good investment states. When continuation values are higher than liquidation values, on the other hand, lenders face a more difficult problem. In this case, as we discuss next, they need to weigh in the likelihood that projects succeed.

When investments are likely to succeed (call it “booms”), the probability that borrowers are in distress is small. Lenders’ payoffs will come mostly from regular debt repayments. To maximize those payoffs, lenders will prefer to take negative net economic ownerships (overinsure with CDS). CDS overinsurance will then maximize debt repayments consistent with borrowers exerting high effort to make their firms profitable. Conditional on distress, however, firms with CDS-overinsured lenders will be promptly liquidated — the empty creditor problem is pronounced in booms.

In “busts,” the probability of investment failure is higher and lenders’ expected payoffs lean more towards outcomes associated with default (out-of-court restructuring and liquidation values). If continuation values are higher than liquidation values, lenders will be inclined to have zero net economic ownership — the empty creditor problem is reduced. In this scenario, zero net economic ownership reduces borrowers’ payoffs when their firms are in distress (as lenders stand to capture restructuring surpluses). This, in turn, prompts borrowers to exert high effort to make their investments more likely to succeed. On the flip side, if

continuation values are low and approximate liquidation values, the gains from renegotiation decline. Lenders will then be more inclined to overinsure. Notably, Because investments are more likely to fail in bad states, this dynamic might lead one to “too often” observe CDS-insured lenders forcing firms with low continuation values into bankruptcy during busts.

The endogenous link between the demand for CDS and the state of the economy described by our model is new to the literature.<sup>7</sup> The implications of a contracting framework that allows for complexities such as commitment and moral hazard problems stand in contrast to the extant notion that CDSs are harmful for allowing lenders to have negative economic ownership in the firms they finance. Additional model analysis shows that, in booms, CDS overinsurance increases financing to levels that *exceed* financing in economies where lender ownership is constrained to be non-negative. In busts, CDSs increase funding to levels that, at a minimum, *equal* those in economies where lender ownership is constrained to be non-negative. Naturally, there are more investment failures in downturns and, observationally, there are more bankruptcies being pushed forward by lenders that are CDS insured during those times. In the absence of a benchmark, however, that casual observation is uninformative about the role played by CDSs in busts.

The theory we propose has several empirical implications and sheds light on recent attempts to find evidence on the empty creditor hypothesis. We show, for example, that CDSs are more beneficial for firms that are safer and have higher continuation values. This result is surprising as one might expect riskier firms to benefit the most from the existence of CDS

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<sup>7</sup>In the existent literature, Bolton and Oehmke (2011) do not consider the impact of investment success on the demand for CDS (CDS contracting in their paper ignores the state of the economy and managerial effort), while Arping (2004) does not consider the incentives of lenders to overinsure with CDS.

insurance. Consistent with our model, recent studies by Ashcraft and Santos (2009) and Hirtle (2009) find that safer, larger firms have benefited the most from CDS contracts (for example, by paying lower interest on their bank loans once CDSs are written on their bonds). Our theory predicts that the beneficial effects of CDSs on firm financing are present even when aggregate credit is tight. Consistent with this prediction, Saretto and Tookes (2011) find that CDSs increased corporate leverage and debt maturity even during the 2008–9 crisis. Our model also implies that the empty credit problem is procyclical; that is, the conditional probability of CDS-led liquidation given that a firm is in distress is higher (lower) in booms (busts). As such, the model reconciles results from empirical studies looking at the role of CDSs in influencing the choice between restructuring and bankruptcy during recent contractions, including the financial crisis (e.g., Mengle (2009), Aspeli and Iden (2010), and Bedendo et al. (2010)).

Our analysis has direct implications for the debate about the optimal regulation of CDS markets. Hu and Black (2008a,b) argue that voting in restructuring decisions should be limited to lenders with positive net economic ownerships. We argue, in turn, that this constraint would destroy the ex ante benefits of CDSs. Bolton and Oehmke (2011) suggest that eliminating negative net economic ownerships (CDS overinsurance) would increase efficiency since this would reduce the risk of breakdowns in restructurings. In contrast to their recommendation, we show that banning overinsurance is undesirable in a world where economic fluctuations affect investment prospects. Indeed, we show that imposing strict limits on CDS positions may end up reducing firms' credit capacity when they most need it.

Our paper is related to an infant literature on links between CDSs and creditor–borrower

relations. Most papers in this literature focus on the effect of CDSs on adverse selection and moral hazard problems. Duffee and Zhou (2001) show that CDSs can alleviate “lemons problems” in credit risk-transfer markets. Parlour and Winton (2008) show how loan sales and CDSs might jointly emerge in equilibrium, characterizing risk-transfer efficiency. Parlour and Plantin (2008) further investigate the effect of CDS markets on banks’ incentive to monitor (see also Morrison (2005)). Arping (2004) shows that CDSs increase the commitment of lenders to terminate projects in the presence of moral hazard. Bolton and Oehmke (2011) show that CDSs help reduce strategic renegotiations. Creditors in their model disregard payoffs associated with non-default states and optimal CDS insurance is independent of investment prospects. The aforementioned papers are silent on the interplay between the empty creditor problem, the probability of investment success (state of the economy and effort), and the demand for CDS.

The remainder of the paper is organized as follows. Section 2 describes the model setup. In Section 3, we analyze the consequences of CDS contracting on renegotiation and liquidation outcomes. Section 4 characterizes the interplay between debt repayment, CDS contracts, and borrowers’ effort choices. Section 5 derives the demand for CDS. In Section 6, we characterize the efficiency of CDS markets and study welfare implications of imposing constraints on those markets. We present a set of empirical implications in Section 7. Section 8 concludes the paper. All proofs are in Appendix C.

## 2.2 Model Setup

There are three risk neutral players: a borrower, a lender, and a competitive CDS provider. The game is played in three periods  $t = 0, 1, 2$ . The borrower is penniless, but endowed with a project. He turns to a lender to fund the project.

The time line and structure of the game is depicted in Figure 2.1. The project needs  $I > 0$  units of investment in  $t = 0$ . If a project receives investment in  $t = 0$ , it generates outcome  $o_1 \in \{0, y_1\}$  in  $t = 1$ . Following Hart and Moore (1994, 1998) and Bolton and Scharfstein (1990, 1996), we assume that  $o_1$  is non-verifiable. If the lender decides to finance the project, she makes a take-it-or-leave-it offer to the borrower. A contract specifies a repayment  $\tilde{R}_1(\tilde{o}_1)$  to be made to the lender in  $t = 1$ , where  $\tilde{o}_1 \in \{0, y_1\}$  is the outcome reported by the borrower. We assume limited liability such that  $\tilde{R}_1(\tilde{o}_1) \leq \tilde{o}_1$ . Accordingly, we have  $\tilde{R}_1(0) = 0$ , and a contract is characterized by  $R_1 \equiv \tilde{R}_1(y_1)$ .

[Figure 2.1 about here]

If the borrower accepts the offer, he chooses his effort level. The borrower chooses either to exert high effort,  $e_H$ , or low effort,  $e_L$ . The distribution of the short-term outcome  $o_1$  depends on the effort level. The probability that  $o_1 = y_1$  is  $p_H$  if the borrower chooses  $e = e_H$ , and  $p_L$  if the borrower chooses  $e = e_L$ , where  $p_H > p_L$ . If the borrower chooses  $e_L$ , he derives a private benefit  $B > 0$ . Effort choices are not observed by the lender, who has belief  $\mu$  that the borrower chooses  $e = e_H$ . After the borrower decides on his effort, the lender decides whether to buy a CDS. If the lender buys a CDS, she chooses the repayment that accrues if a “credit event” occurs in  $t = 1$ , and pays the correspondent fee  $f$  to the CDS

provider. We model the payment received by the lender in the event of liquidation according to practice in the CDS market. The lender retains the liquidation value of the investment (interpreted as proceeds from Chapter 11)  $\beta I$ , where  $0 < \beta < 1$ . In addition, she also receives the compensation amount  $\pi$ . Since the CDS market is competitive, the premium  $f$  is fairly priced. A credit event is said to occur if the borrower is formally in default; i.e., if the borrower reports  $\tilde{o}_1 = 0$  and the lender refuses to engage in a voluntary debt renegotiation.

At the beginning of  $t = 1$ , outcome  $o_1$  is realized. If the borrower reports  $\tilde{o}_1 = y_1$ , then no default occurs. In this case, the project continues and generates outcome  $o_2 = y_2$  in  $t = 2$ .<sup>8</sup> The borrower's payoff is  $y_1 - R_1 + y_2$  and the lender receives  $R_1$ . We assume that  $y_2$  cannot be contracted upon ex ante, but it can be verified by the lender in  $t = 2$ . Following Calomiris and Kahn (1991) and Krasa and Villamil (2000), we assume that the verification technology is imperfect, that is, an amount  $(1 - \delta)y_2$  of the continuation outcome cannot be verified by an outside court and remains with the borrower. In this case, the lender can verify  $\delta y_2$  at a cost  $(1 - \lambda)\delta y_2$ .

If the borrower reports  $\tilde{o}_1 = 0$ , the lender can either engage in renegotiation or force liquidation. If the lender refuses to renegotiate, the borrower defaults on his debt. The project is liquidated and the lender receives  $L(\pi) \equiv \beta I + \pi$ , while the borrower receives  $o_1$ . If the lender adheres to a renegotiation schedule, both the lender and the borrower bargain over the value  $\tilde{y}_2 \equiv \lambda \delta y_2$  in  $t = 2$ . In this case, the lender receives  $x$  and the borrower receives  $o_1 + (1 - \delta)y_2 + \tilde{y}_2 - x$ , where  $x$  is the outcome of the renegotiation game.

The borrower may choose to strategically renegotiate (i.e., to trigger renegotiation even

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<sup>8</sup>In principle, there could be uncertainty regarding the continuation outcome  $o_2$  (as in Bolton and Oehmke (2011)). Our results, however, are qualitatively similar in the presence of uncertainty.



when  $o_1 = y_1$ ) if the lender cannot credibly commit to liquidate and the borrower's payoff under renegotiation is sufficiently high. However, CDSs increase the lender's payoff under liquidation, which makes the lender's threat to liquidate more credible. As we show below, CDSs also increase the lender's bargaining position under renegotiation, hence her share of the project's continuation value  $y_2$ . The amount of CDS insurance bought by the lender can be used as a way of strengthening her bargaining power when the project is not liquidated, or as a liquidation commitment device.

## 2.3 CDS, Renegotiation, and Default

We start our equilibrium analysis by investigating the outcome that would prevail when the borrower triggers renegotiation ( $\tilde{o}_1 = 0$ ) and the lender accepts to renegotiate. We use a standard Nash-bargaining solution concept where the borrower and the lender disagreement payoffs are 0 and  $L(\pi)$ , respectively. According to this concept, the bargaining outcome will be given by

$$x(L(\pi)) = \frac{1}{2}\tilde{y}_2 + \frac{1}{2}L(\pi). \quad (2.1)$$

From equation (3.1) one can see that the lender's (gross) *economic ownership* — her share of the continuation value — is increasing in both the amount of CDS protection  $\pi$  and the liquidation value  $\beta I$ .<sup>9</sup>

For the purpose of our analysis, we assume that  $x(L(0)) > L(0)$ . This implies that the lender prefers renegotiation to liquidation and that liquidation is ex post inefficient.

Given the outcome of renegotiation, the lender refuses to renegotiate if  $L(\pi) > x(L(\pi))$

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<sup>9</sup>Hu and Black (2007) define economic ownership as “...the economic return on shares, which can be achieved directly by holding shares or indirectly by holding a coupled asset.”

and engages in renegotiation if  $L(\pi) \leq x(L(\pi))$ . We note that an increase in  $L(\pi)$  not only directly affects the threat point of liquidation (one-to-one), but also the renegotiation outcome (also in a linear fashion, but with slope smaller than one). Therefore, there exists a unique threshold of  $L(\pi^*)$  such that the lender is indifferent between liquidation and renegotiation. We assume that the lender renegotiates if she is indifferent between the payoff of renegotiation and that of bankruptcy. This leads to the following proposition.

**Proposition 1** *Suppose the borrower triggers renegotiation. The lender refuses to renegotiate if  $L(\pi) > L(\pi^*)$  and engages in renegotiation if  $L(\pi) \leq L(\pi^*)$ , where  $L(\pi^*) = \tilde{y}_2$ .*

Proposition 1 says that the lender's maximum payoff consistent with renegotiation is attained when she buys credit protection in the amount of  $\pi = \pi^*$ . Although CDS protection above  $\pi^*$  increases the lender's economic ownership, it reduces her interest to renegotiate and results in default. The reason is that the lender's incentive to renegotiate is dictated by her *net* economic ownership. The lender's net economic ownership is a combination of her share of the continuation value and her payoff under bankruptcy.<sup>10</sup> Accordingly, credit protection above  $\pi^*$  builds up *negative net economic ownership*, while credit protection below that amount results in *positive net economic ownership*. If the lender's CDS protection is equal to  $\pi^*$ , she has *zero net economic ownership*.

For credit protection values at most as high as  $\pi^*$ , the lender builds up non-negative net economic ownership and engages in renegotiation. In this case, the maximum renegotiation payoff is given by  $\tilde{y}_2$ . Credit protection in excess of  $\pi^*$  results in negative net economic ownership, in which case the lender refuses to renegotiate and forces the borrower into bankruptcy.

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<sup>10</sup>Hu and Black (2007) define net economic ownership as "...a person's combined economic ownership of host company shares and coupled assets, and can be positive, zero, or negative."

We now derive the borrower's decision to call for strategic renegotiation given that the project succeeds ( $o_1 = y_1$ ). If the borrower reports the truth (i.e.,  $\tilde{o}_1 = y_1$ ), his payoff is  $y_1 - R_1 + y_2$ . This payoff needs to be compared to that when he lies ( $\tilde{o}_1 = 0$ ). In this case, if the lender renegotiates, the borrower's payoff is  $y_1 + (1 - \delta)y_2 + \tilde{y}_2 - x(L(\pi))$ . If the lender does not renegotiate, the borrower's payoff is  $y_1$ . We assume that the borrower does not call for renegotiation when he is indifferent between diverting the realized cash flow and reporting the true outcome.

Intuitively, the borrower triggers strategic renegotiation if the face value of debt  $R_1$  exceeds a threshold. The game is solved backwards and if the lender is expected to renegotiate, the borrower triggers renegotiation if  $R_1 > \delta y_2 - \tilde{y}_2 + x(L(\pi))$ . If the lender is expected to liquidate, the borrower triggers renegotiation if  $R_1 > y_2$ . We summarize this result in the following proposition.

**Proposition 2** *Suppose the project succeeds. Then:*

- (1) *If the lender has non-negative net economic ownership, the borrower triggers renegotiation if and only if  $R_1 > \delta(1 - \lambda)y_2 + x(L(\pi))$ .*
- (2) *If the lender has negative net economic ownership, the borrower triggers renegotiation if and only if  $R_1 > y_2$ .*

Proposition 2 states that, as long as the lender has positive net economic ownership, i.e.,  $\pi < \pi^*$ , an increase in  $\pi$  (and hence  $L(\pi)$ ) reduces the borrower's incentive to strategically trigger renegotiation. In other words, an increase in the amount of CDS protection continuously increases the threshold value for repayment  $R_1$ . At  $\pi = \pi^*$  (zero net economic ownership), the threshold value for  $R_1$  hits a discontinuity as the lender's economic ownership becomes negative. The proposition implies that one needs to consider only the following

two cases in the analysis of optimal CDS demand: (1) the lender has zero net economic ownership, i.e.,  $\pi = \pi^*$ ; or (2) the lender has negative net economic ownership, i.e.,  $\pi > \pi^*$ . By choosing the amount of CDS protection,  $\pi$ , the lender modulates her net economic ownership in the firm. Our paper is the first to demonstrate how this choice is optimally made.

## 2.4 Debt Repayments, CDS, and Effort

In the last section we identified the key tradeoff faced by the lender in our CDS model. Although overinsurance allows the lender to receive higher debt repayments when investment is successful, it comes at the cost of triggering bankruptcy when investment fails. Bankruptcy gives the lender the liquidation value  $\beta I$ , which is smaller than her share of the continuation value under renegotiation,  $\delta \lambda y_2$ . An important factor influencing this tradeoff is the borrower's effort choice. In this section we study how the lender chooses an optimal level of credit protection given that she does not observe the borrower's effort choice. We then analyze how the lender's choice of debt repayment affects the equilibrium of this CDS–Effort subgame.

### 2.4.1 The CDS–Effort Subgame

In this subsection we characterize the equilibria of the CDS–Effort subgame. First, we derive the lender's optimal CDS demand given her expectation regarding the borrower's effort choice. Next, we determine the borrower's best effort choice given the lender's choice of CDS protection. Finally, we combine both the lender's and the borrower's best responses in order to find the equilibria.

If the borrower chooses high effort, he increases the probability that the project suc-

ceeds. This weights the lender's expected payoff more towards the repayment  $R_1$ , making CDS-overinsurance more attractive given the greater bargaining power this position entails. The problem is that the lender does not observe the borrower's effort level and must make her decision on the amount of CDS under uncertainty. Proposition 3 characterizes the lender's optimal decision regarding the level of CDS protection.

**Proposition 3** *The lender's net economic ownership is determined as follows:*

(1) *For  $R_1 > y_2$ , the lender chooses to have zero net economic ownership.*

(2) *For  $R_1 \in (\delta y_2, y_2]$ ,*

*(i) the lender chooses to have negative net economic ownership if*

$$R_1 > R(\mu) \equiv \frac{\delta \lambda y_2 - \beta I [\mu(1 - p_H) + (1 - \mu)(1 - p_L)]}{\mu p_H + (1 - \mu) p_L},$$

*(ii) the lender chooses to have zero net economic ownership if  $R_1 \leq R(\mu)$ .*

(3) *For  $R_1 \leq \delta y_2$ , the lender chooses to have zero net economic ownership.*

According to Proposition 3, the lender does not overinsure if she chooses  $R_1 > y_2$ . The reason is that a repayment  $R_1 > y_2$  causes the borrower to trigger strategic renegotiation, implying that renegotiation takes place independent of the outcome  $\omega_1$ . If the lender overinsures, she refuses to renegotiate and the borrower defaults. The payoff of the lender is  $\beta I$ . If the lender does not build up negative economic ownership, her payoff is  $\tilde{y}_2 > \beta I$ . Therefore, the lender is better off without overinsurance.

If the lender chooses  $R_1 \in (\delta y_2, y_2]$ , then Proposition 3 shows that overinsurance is attractive for the lender provided that she is able to receive a high debt repayment. In this case, if the lender chooses  $\pi = \pi^*$ , renegotiation is always triggered and the lender's payoff is given

by  $\delta\lambda y_2$ . If the lender overinsures, she receives  $R_1$  when the project succeeds and receives  $\beta I$  when the project fails. Given the lender's expected probability of success  $\mu p_H + (1 - \mu) p_L$ , the repayment  $R_1$  must be high enough so as to compensate for foregone renegotiation proceeds. This translates into the requirement that  $R_1 > R(\mu)$ . Since  $R(\mu)$  is decreasing in  $\mu$ , the more the lender believes the borrower is exerting low effort, the higher the repayment must be in order to compensate her for the loan. Another important observation is that the higher the recovery value  $\beta I$ , the smaller the repayment necessary to induce overinsurance.

A repayment  $R_1 \leq \delta y_2$  is insufficient to induce overinsurance. In this situation, zero net economic ownership is enough to avoid strategic renegotiation. Accordingly, overinsurance only decreases the lender's payoff since foregone renegotiation proceeds are higher than bankruptcy proceeds.

Proposition 3 described the lender's best choices of CDS insurance given her beliefs about the borrower's effort choice. To find the equilibria of this subgame, we need to derive the borrower's effort choices given the lender's amount of insurance. This is given by Proposition 4. We assume that whenever the borrower is indifferent between exerting high effort and low effort, he chooses the former.

**Proposition 4** *Let  $\Delta \equiv y_1 - \frac{B}{(p_H - p_L)}$ . The borrower's choice of effort is determined as follows:*

- (1) *For  $R_1 > y_2$ , the borrower chooses high effort if and only if  $y_1 (p_H - p_L) \geq B$ .*
- (2) *For  $R_1 \in (\delta y, y_2]$ ,*
  - (i) *if the lender has negative net economic ownership, the borrower chooses high effort if and only if  $\bar{R}_1 \equiv y_2 + \Delta \geq R_1$ ,*
  - (ii) *if the lender has non-negative net economic ownership, the borrower chooses high effort if and only if  $y_1 (p_H - p_L) \geq B$ .*

(3) For  $R_1 \leq \delta y_2$ , the borrower chooses high effort if and only if

$$\underline{R}_1 \equiv \delta y_2 + \Delta \geq R_1.$$

We assume that it is optimal to implement high effort in the absence of strategic default and liquidation given default. In this efficient world, investment  $I$  should be made if and only if

$$\begin{aligned} \Pi \equiv \max \{ & p_H (y_1 + y_2) + (1 - p_H) [(1 - \lambda) y_2 + \tilde{y}_2] \\ & , p_L (y_1 + y_2) + (1 - p_L) [(1 - \lambda) y_2 + \tilde{y}_2] + B \} \geq I. \end{aligned} \quad (2.2)$$

Conditional on the project having positive NPV, i.e.,  $\Pi > 0$ , high effort should be induced if and only if  $y_1 (p_H - p_L) + \delta (1 - \lambda) y_2 \geq B$ . Along with this assumption, we also assume that  $y_1 (p_H - p_L) < B$ .<sup>11</sup> These two assumptions imply that verification costs are sufficiently high such that it is optimal to induce high effort in order to avoid those costs.

It follows from Proposition 4 that, if the lender has zero net economic ownership, the borrower's compensation in the event the project succeeds must be sufficiently high to induce him to exert high effort. Alternatively, if the lender chooses a debt repayment that is sufficiently high, then she must build up negative net economic ownership to induce high effort. If the lender overinsures, she can credibly threaten to reject renegotiation and force the borrower into bankruptcy. This reduces the borrower's payoff when investment fails and creates a compensation scheme that induces high effort.

Proposition 5 characterizes the equilibria of the CDS–Effort subgame. While the proposi-

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<sup>11</sup>If we assume otherwise, then according to Proposition 4 it would follow that high effort is always implemented.

tion seems fairly involved, it reveals a number of economically interesting results. We discuss them in turn.

**Proposition 5** *Let  $R(0) \in (\delta y_2, y_2)$ . The equilibria of the CDS–Effort subgame are determined as follows:*

- (1) *For  $R_1 > y_2$ , the lender chooses zero net economic ownership and the borrower chooses low effort.*
- (2) *Let  $R_1 \in (\delta y_2, y_2]$ .*
  - (i) *For  $R_1 \in (R(0), y_2]$ : (a) if  $R_1 > \overline{R}_1$ , the lender chooses negative net economic ownership and the borrower chooses low effort; (b) if  $R_1 \leq \overline{R}_1$ , the lender chooses negative net economic ownership and the borrower chooses high effort.*
  - (ii) *For  $R_1 \in (\delta y_2, R(0)]$ : (a) if  $R_1 > \overline{R}_1$ , the lender chooses zero net economic ownership and the borrower chooses low effort; (b) if  $R_1 \leq \overline{R}_1$ , there is one equilibrium in which the lender chooses negative net economic ownership and the borrower chooses high effort, and another in which the lender chooses zero net economic ownership and the borrower chooses low effort.*
- (3) *For  $R_1 \leq \delta y_2$ :*
  - (i) *if  $R_1 > \underline{R}_1$ , the lender chooses zero net economic ownership and the borrower chooses low effort;*
  - (ii) *if  $R_1 \leq \underline{R}_1$ , the lender chooses zero net economic ownership and the borrower chooses high effort.*

If the lender charges a repayment that is too high (i.e.,  $R_1 > y_2$ ), then she chooses to have zero net economic ownership. According to Proposition 5, this debt repayment is insufficient to induce the borrower to exert high effort. If the lender chooses a debt repayment below  $R_1 \leq \delta y_2$ , then she also prefers a zero net economic ownership position. The reason is that the lender receives  $\delta R_1$  when the project succeeds under both negative and zero net economic ownerships. On the other hand, if the lender overinsures and the project fails, she receives



the liquidation value  $\beta I$ , as opposed to the renegotiation surplus  $\tilde{y}_2 > \beta I$ . The borrower's effort choice depends on his payoff when the project succeeds. If the repayment chosen by the lender is above  $\underline{R}_1$ , then the borrower exerts low effort and derives benefit  $B$ . If the lender's repayment is sufficiently low ( $R_1 \leq \underline{R}_1$ ), then the borrower has enough incentives to choose high effort in order to increase the project's probability of success.

The analysis is slightly more involved when the lender's choice of debt repayment lies in the interval  $(\delta y_2, y_2]$ . If the debt repayment is sufficiently high ( $R_1 > R(0)$ ), the lender prefers to overinsure. This result follows from the fact that the debt repayment received when the project succeeds is large enough so as to compensate for the foregone renegotiation proceeds when the project fails. If the debt repayment is such that  $\overline{R}_1 < R_1 \leq R(0)$ , the lender chooses zero net economic ownership. The intuition for nonexistence of an equilibrium with overinsurance is as follows. Proposition 4 says the lender overinsures if and only if her belief that the borrower chooses high effort is sufficiently high, such that  $R_1 > R(\mu)$ . Proposition 5 says that for  $R_1 > \overline{R}_1$ , the borrower's optimal choice of effort is  $e_L$ , which implies that the lender's updated belief is  $\mu = 0$ . However, because  $R_1 \leq R(0)$  the lender is better off without overinsurance.

If  $R_1 \leq \overline{R}_1 \leq R(0)$ , then there are two equilibria. On the one hand, if the lender anticipates that the borrower will choose high effort (i.e.,  $R_1 > R(\mu)$ ), then according the Proposition 4 she overinsures. From Proposition 5 we know that overinsurance induces the borrower to exert high effort, which results in  $\mu = 1$ . This reinforces the lender's willingness to assume a negative net economic ownership as  $R_1 > R(\mu) \geq R(1)$ . On the other hand, if the lender anticipates the borrower will choose low effort, then  $R_1 \leq R(\mu)$ . Proposition 4

says that the lender chooses to have zero net economic ownership and Proposition 5 implies that the borrower chooses low effort. Accordingly, the lender's updated belief is  $\mu = 0$ , which confirms her decision to not overinsure since  $R_1 \leq R(\mu) \leq R(0)$ .

Proposition 5 shows, in essence, that the lender can choose a debt repayment schedule from two different sets. If she chooses a repayment from the set of low values, then just-insurance is enough to avoid strategic default. On the other hand, to achieve the same outcome when choosing from the set with high values, she must overinsure. Regardless of the set from which the lender chooses the repayment, she needs to select a value that is sufficiently small (within the relevant range) in order to induce high effort.

### 2.4.2 Debt Repayment and Effort

The analysis in the preceding subsection shows the possibility for equilibria with both zero and negative net economic ownership in the CDS–Effort subgame for  $R_1 \in (\delta y_2, y_2]$ . In particular, equilibria with zero net economic ownership occur when  $R_1 \in (\delta y_2, R(0)]$ . However, the lender chooses a repayment in this range only if  $R_1 \leq \bar{R}_1$  and the equilibrium played is one that results in overinsurance. This result follows from the fact that if the equilibrium for  $R_1 \in (\delta y_2, R(0)]$  involves zero net economic ownership and low effort, then a repayment in this range is strictly dominated by a repayment of  $R_1 = \delta y_2$ . To see this point, note that if the lender chooses  $R_1 \in (\delta y_2, R(0)]$ , then renegotiation is always triggered and her payoff is  $\tilde{y}_2$ . If the lender chooses  $R_1 = \delta y_2$ , then the borrower does not call for strategic renegotiation and the lender's payoff is

$$\Pi(\delta y_2) \equiv [p_L \delta + (1 - p_L) \delta \lambda] y_2, \quad (2.3)$$

which is strictly greater than  $\tilde{y}_2$ .

As a consequence of the preceding analysis, the lender chooses  $R_1 \in (\delta y_2, y_2]$  only if she overinsures. The lender's dilemma within this range is whether to choose a low repayment  $R_1 \leq \overline{R}_1$  consistent with high effort, or require a high repayment  $R_1 > \overline{R}_1$  at the expense of inducing low effort. To streamline our subsequent analysis, we assume that the equilibrium played in (ii)(b) of Proposition 5 is the one that results in negative net economic ownership.<sup>12</sup> We are then able to characterize the lender's debt repayment choice.

If the lender chooses  $R_1 \leq \delta y_2$ , then she faces a tradeoff between (1) requiring a repayment that is consistent with the borrower exerting high effort, and (2) demanding a higher repayment that induces the borrower to choose low effort and increases the probability of failure and renegotiation. If the lender chooses the former, her payoff is given by:

$$\Pi(\underline{R}_1) \equiv p_H \Delta + [p_H \delta + (1 - p_H) \delta \lambda] y_2. \quad (2.4)$$

If the lender chooses  $R_1 \in (\delta y_2, y_2]$ , she faces a similar tradeoff. In particular, when the lender chooses  $R_1 = y_2$ , her payoff is

$$\Pi(y_2) \equiv (1 - p_L) \beta I + p_L y_2, \quad (2.5)$$

while if she chooses  $R_1 = \overline{R}_1$ , her payoff is

$$\Pi(\overline{R}_1) \equiv p_H \Delta + (1 - p_H) \beta I + p_H y_2. \quad (2.6)$$

Proposition 6 derives the conditions under which the lender chooses to induce high effort.

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<sup>12</sup>If the equilibrium played is the one that results in zero net economic ownership, then if  $\overline{R}_1 \leq R(0)$ , the lender chooses  $R_1 \in (\delta y_2, y_2]$  only if  $R_1 > \overline{R}_1$  (i.e., only if she induces low effort).

**Proposition 6** *The level of effort induced by the lender is determined as follows:*

(1) *For  $R_1 \in (\delta y_2, y_2]$ , the lender chooses  $R_1 = \overline{R_1}$  over  $R_1 = y_2$  if and only if*

$$y_2 \geq \underline{y_2} \equiv -\frac{p_H}{p_H - p_L} \Delta + \beta I.$$

(2) *For  $R_1 \leq \delta y_2$ , the lender chooses  $R_1 = \underline{R_1}$  over  $R_1 = \delta y_2$  if and only if*

$$y_2 \geq \overline{y_2} \equiv -\frac{p_H}{\delta(p_H - p_L)(1 - \lambda)} \Delta.$$

Proposition 6 shows that, within a given range, it is optimal for the lender to induce high effort if and only if the project's continuation value is sufficiently large. Since there is no strategic renegotiation in equilibrium, the lender's payoff is partially dependent upon the debt repayment received when the project succeeds. Recall, Proposition 2 showed that the higher the project's continuation value, the higher the debt repayment consistent with no strategic renegotiation. The probability that the project succeeds is thus partially dependent upon the effort that the borrower chooses. In order for the lender to increase the probability of success, she must give up some debt repayment to induce the borrower to exert high effort.

At the same time, increases in the project's continuation value improve the tradeoff terms in favor of inducing high effort. A higher probability of success makes the lender's payoff more sensitive to the continuation value. As Proposition 4 showed, higher continuation values increase the debt repayment consistent with the borrower exerting high effort. Proposition 6 shows that for large enough continuation values, the lender prefers to induce the borrower to exert high effort.

## 2.5 CDS Contracts and the Economy

Although Proposition 6 describes the lender's tradeoff between payoffs associated with debt repayment and investment failure, it does not shed light on the lender's choice to have zero or negative net economic ownership — the demand for CDS. We describe this choice in turn.

The lender overinsures if she chooses a debt repayment in the range  $(\delta y_2, y_2]$ , and assumes a zero net economic ownership if she chooses a debt repayment such that  $R_1 \leq \delta y_2$ . Proposition 7 introduces one of the central results of our paper. It characterizes the lender's optimal repayment choice when the liquidation value is sufficiently small. A more complete characterization is given in the Appendix.

**Proposition 7** *Let  $c(p) = \delta\lambda + \delta(1 - \lambda)p$ . If  $\beta I$  is sufficiently small, the lender's repayment choice is characterized by cutoffs  $y_2^* < \underline{y}_2 < y_2^{**} < \overline{y}_2 \leq y_2^{***}$  such that:*

(1) *If  $p_H > p_L \geq c(p_H) > c(p_L)$ , the lender chooses*

$$R_1 = \begin{cases} y_2, & \text{for } y_2 < \underline{y}_2 \\ \overline{R}_1, & \text{for } y_2 \geq \underline{y}_2 \end{cases}.$$

(2) *If  $p_H \geq c(p_H) > c(p_L) > p_L$ , the lender chooses*

$$R_1 = \begin{cases} y_2, & \text{for } y_2 < y_2^* \\ \delta y_2, & \text{for } y_2 \in [y_2^*, y_2^{**}) \\ \overline{R}_1, & \text{for } y_2 \geq y_2^{**} \end{cases}.$$

(3) *If  $c(p_H) > p_H > c(p_L) > p_L$ , the lender chooses*

$$R_1 = \begin{cases} y_2, & \text{for } y_2 < y_2^* \\ \delta y_2, & \text{for } y_2 \in [y_2^*, y_2^{**}) \\ \overline{R}_1, & \text{for } y_2 \in [y_2^{**}, y_2^{***}) \\ \underline{R}_1, & \text{for } y_2 \geq y_2^{***} \end{cases}.$$

(4) If  $c(p_H) > c(p_L) \geq p_H > p_L$ , the lender chooses

$$R_1 = \begin{cases} y_2, & \text{for } y_2 < y_2^* \\ \delta y_2, & \text{for } y_2 \in [y_2^*, \bar{y}_2) \\ \underline{R}_1, & \text{for } y_2 \geq \bar{y}_2 \end{cases} .$$

Proposition 7 describes an important tradeoff faced by the lender in our model. If the lender chooses to have negative net economic ownership, she refuses to renegotiate in default and forces the borrower into bankruptcy. This maximizes debt repayments when investment is successful, but reduces the payoff to the liquidation value of the project when the borrower enters distress. If the lender chooses to have zero net economic ownership, she gives up some debt repayment in the event investment succeeds in exchange for maximum renegotiation proceeds in the event it fails. These dynamics are determined by the probability of investment success  $p_H$  and  $p_L$  and project's continuation value.

The first result from Proposition 7 is illustrated in Figure 2.2. If  $p_H > p_L \geq c(p_H) > c(p_L)$ , then the lender's payoff is weighted more towards outcomes associated with investment success (a portion of the project's cash flows). As a result, the extra debt repayment extracted when the project succeeds compensates for the forgone renegotiation proceeds when the project fails.

[Figure 2.2 about here]

The second result from Proposition 7 is described in Figure 2.3. When economic conditions are such that  $p_H \geq c(p_H) > c(p_L) > p_L$ , there is a range of continuation values for which the lender prefers not to overinsure. If the continuation value is low, the tradeoff faced by the lender disappears. The lender's payoffs when the project fails are approximately

the same regardless of her net economic ownership. Therefore, the lender overinsures to maximize her payoff (debt repayment) in the event the project succeeds. In addition, the continuation value is insufficient for high effort to be optimal and the resulting probability of success is  $p_L$ . As the continuation value increases, the opportunity cost of overinsurance also increases. Since the probability of success is still relatively low, expected forgone proceeds from renegotiation are sizeable. Accordingly, it becomes optimal for the lender to have zero net economic ownership. On the other hand, if the continuation value is sufficiently high, then inducing the borrower to exert high effort is attractive for the lender. In this case, overinsurance becomes optimal as it increases the debt repayment consistent with high effort.

[Figure 2.3 about here]

Another implication of Proposition 7 is that overinsurance is more likely to be associated with firms that are safer (higher probability of investment success) and larger (higher continuation values). Along with the fact that the main difference between CDS and standard insurance is that CDS allows for protection beyond economic interest, Proposition 7 helps characterize the types of CDS positions we often observe: CDS are written on safer, larger firms and many times leave lenders “overinsured” in their exposures to firms they lend to.

The results described above suggest that CDS-overinsurance is a likely phenomenon during booms. To wit, higher probabilities of successful investments strengthen the beneficial effects of CDSs on limited commitment and agency problems. This boosts the income that can be pledged to lenders, increasing firm debt capacity. As a result, a larger number of projects with positive NPV receive financing. Because the probabilities of investment failure

are smaller in booms, the *appearance* of the empty creditor problem is reduced during those times. Proposition 7 also shows that the lender prefers to overinsure when continuation values are low. On the other hand, if  $c(p_H) > c(p_L) \geq p_H > p_L$ , the lender chooses to have zero net economic ownership for continuation values that are large. The reason is that, in this case, the lender's payoff is weighted more towards outcomes associated with project failure. This increases the expected forgone renegotiation proceeds and the opportunity cost under negative net economic ownership.

The fourth result of Proposition 7 is depicted in Figure 2.4. When the continuation value is sufficiently low, the lender's payoff when the project fails is the same regardless of her net economic ownership. Thus, the lender prefers overinsurance in order to maximize her payoff in the event the project succeeds. Since the continuation value is low, inducing high effort is not optimal for the lender. Increasing the continuation value raises the attractiveness of inducing the borrower to exert high effort. As a result, the lender chooses to overinsure and to induce high effort. At high continuation values, in contrast, the tradeoff between debt repayment and renegotiation proceeds faced by the lender is sizeable. Because the probability of success is low, the opportunity cost of overinsurance is large and it becomes optimal for the lender to have zero net economic ownership.

[Figure 2.4 about here]

These results suggest that, during busts, CDS overinsurance emerges where credit constraints are most likely to bind. Indeed, CDS overinsurance eases the financing of profitable projects with relatively low continuation values, projects that would likely be underfunded (“financially constrained”) in tight credit markets without CDS contracts. On the flip side, the



lender does not overinsure in busts when project continuation values are high. This optimal CDS policy reduces the empty creditor problem exactly when its drawback is potentially sizeable; that is, when the probability of distress is high. Importantly, zero net economic ownership does not come at the cost of leaving profitable firms underfunded since, in expectation, firms with high continuation values would likely receive funding even in the absence of CDS overinsurance.

Putting these results together, our analysis reveals the striking result that the empty creditor problem is *procyclical*. Although CDS overinsurance leads to bankruptcy when the borrower is distressed, the incidence of overinsurance is higher in booms, when the probability of distress is small. Notably, the dynamics of the demand for CDS over the business cycle works so as to minimize the empty creditor problem. As we discuss below, this result makes the task of finding empirical evidence of the empty creditor problem particularly challenging.

## 2.6 Efficiency and Regulatory Constraints on CDS Markets

From a welfare standpoint, it is important to characterize the efficiency gains associated with the existence of CDS contracts. It is also important to understand how constraints on CDSs — in especial, constraints on CDS overinsurance — may affect credit markets and firms. The analysis of this section considers these issues and sheds light on the economic effects of proposed regulatory changes in CDS markets. The comparison benchmark we use is the equilibrium that obtains in the absence of CDSs.

### 2.6.1 Equilibrium without CDS Markets

Proposition 2 shows that in the absence of CDSs the maximum repayment consistent with no strategic default is given by  $R_1 = \delta (1 - \lambda) y_2 + x(L(0))$ . In order to implement high effort, the repayment chosen by the lender must be such that

$$p_H (y_1 - R_1 + y_2) + (1 - p_H) [(1 - \delta) y_2 + \tilde{y}_2 - x(L(0))] \geq$$

$$p_L (y_1 - R_1 + y_2) + (1 - p_L) [(1 - \delta) y_2 + \tilde{y}_2 - x(L(0))] + B.$$

At the same time, the lender chooses to induce high effort if and only if

$$p_H [\delta (1 - \lambda) y_2 + x(L(0)) + \Delta] + (1 - p_H) [x(L(0))] \geq$$

$$p_L [\delta (1 - \lambda) y_2 + x(L(0))] + (1 - p_L) [x(L(0))],$$

which holds if and only if  $y_2 \geq \overline{y_2}$ .

It follows that the borrower's effort in the absence of CDSs is similar to the effort level that obtains when CDS overinsurance is not allowed. This result is important in order to examine the efficiency properties of CDSs as well as proposals to cap CDS insurance.

### 2.6.2 Efficiency

In our model, efficiency requires: (1) no strategic renegotiation, (2) no liquidation given default, and (3) implementation of high effort. To see this, suppose the realized outcome is  $o_1 = y_1$ . If the borrower does not call for renegotiation (i.e.,  $\tilde{o}_1 = y_1$ ), then total welfare is  $y_1 + y_2$ . If a strategic renegotiation process takes place ( $\tilde{o}_1 = 0$ ), then total welfare is  $y_1 + (1 - \delta) y_2 + \tilde{y}_2 < y_1 + y_2$ . Accordingly, strategic renegotiation is inefficient. Failure to

renegotiate when  $o_1 = 0$  is also inefficient. Total welfare under renegotiation is  $(1 - \delta) y_2 + \tilde{y}_2$ . However, if the lender refuses to engage in renegotiation, the lender defaults and total welfare is  $\beta I < (1 - \delta) y_2 + \tilde{y}_2$ . Finally, implementation of high effort is efficient under the assumption that  $y_1 (p_H - p_L) + \delta (1 - \lambda) y_2 \geq B$ .

In order to assess the efficiency properties of CDS contracts, we need to examine the equilibrium levels of effort and insurance as functions of the project's continuation value and probability of success. We can use Proposition 7 to compile a table that helps illustrate the problem.

[Table 2.1 about here]

Table 2.1 shows the equilibrium levels of CDS insurance (overinsurance vs. no-overinsurance) and effort (low vs. high) for various combinations of investment success probability and continuation values. Each entry has the CDS–Effort equilibrium outcome that obtains for a continuation value that is lower than the level specified in the column heading. One can readily see from the table that overinsurance is increasing in the probability of investment success. The table also suggests that effort is increasing in the project's continuation value, probability of success, and the level of CDS insurance.

To give context to the results in Table 2.1, recall that in the absence of CDSs,  $\pi = 0$ . In this case, the lender's share of the continuation value resulting from renegotiation is  $x(L(0))$ . This value is smaller than  $\tilde{y}_2$ , which is her share when she just-insures, i.e.,  $\pi = \pi^*$ . The borrower receives a greater share of the continuation value when he calls for renegotiation, which increases his incentive to strategically default. The maximum debt repayment consistent with no strategic renegotiation is therefore smaller in the absence of CDSs. Because

the effort levels implemented without CDSs and with just-insurance are the same, it follows that it is always efficient for the lender to have zero net economic ownership. CDS (just-)insurance improves debt capacity and does not cause the empty creditor problem.

If the level of effort implemented is the same with and without CDSs, then overinsurance is inefficient if the project can be financed in the absence of CDS contracts. The cause of this inefficiency is the empty creditor problem. Although CDSs increase lenders' payoffs and debt capacity, they also bring the threat of inefficient liquidation. If a project cannot be financed with an amount of  $\pi^*$  of CDS protection, then overinsurance is efficient if it allows the project to be financed.

Within this context, Table 2.1 depicts the efficiency role of CDS insurance; in particular, CDS overinsurance. Despite the fact that overinsurance may lead to the empty creditor problem, in equilibrium, overinsurance is more likely to emerge when the probability of investment success is high (see upper part of Table 2.1). In addition, CDS overinsurance helps implement the efficient level of effort (more often than not, overinsurance is associated with high effort in Table 2.1). Indeed, without CDSs (or when only just-insurance is allowed), high effort is only implemented for continuation values above  $\bar{y}_2$ . Recall, a concern with CDS overinsurance is that losses brought about by the empty creditor problem are increasing in continuation values. However, Table 2.1 shows that this effect is partially offset by the fact that effort is also increasing in continuation values, which reduce the probability of inefficient liquidation.

Finally, note that the inefficiency of empty creditors is higher when the verification cost is lower ( $\lambda$  is higher), which results in higher forgone renegotiation proceeds. However, from Proposition 7 one can see that the cutoffs  $c(p^*)$ ,  $c(p_H)$ , and  $c(p_L)$  are increasing in  $\lambda$ . This

makes the equilibria depicted in the lower part Table 2.1 more likely to obtain, implying less overinsurance.<sup>13</sup>

### 2.6.3 Constraints on CDS

According to the analysis of the last subsection, it is efficient for the lender to have zero net economic ownership. This result questions the reform proposals made by Hu and Black (2008a,b), who argue that lenders' CDS positions should be limited to positive net economic ownerships. Under that proposed reform, our model says that restructuring proceeds would be inefficiently reduced when zero net economic ownership is optimal.

If the level of effort implemented under both zero and negative net economic ownerships are the same, then overinsurance is inefficient if the project can be financed with just-insurance. If this is the case, proposals to restrict net economic ownership to be non-negative, such as Bolton and Oehmke (2011), could increase welfare. However, as pointed by our model, overinsurance minimizes agency problems by allowing the implementation of high effort. When this happens, the gains brought about by overinsurance in terms of higher probability of success can offset the losses caused by the empty creditor problem. Our analysis suggests that banning CDS overinsurance may thus be unwarranted.

To characterize this latter point, we need to start by considering equilibria that result in overinsurance and high effort in the absence of constraints on CDSs. These equilibria must then lead to low effort if we ban CDS overinsurance. These scenarios are described in

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<sup>13</sup>Although not depicted in Table 2.1, note that the expected inefficiency of CDS overinsurance is reduced by a better verification technology (higher  $\delta$ ) and a higher recovery rate ( $\beta$ ). The former implies higher verification costs and reduces the proceeds from renegotiation, while the latter increases the proceeds from liquidation.

Table 2.1 by the outcomes with overinsurance and continuation values above  $\underline{y}_2$  and below  $\overline{y}_2$ . Total welfare with negative net economic ownership is given by

$$W_- \equiv p_H (y_1 + y_2) + (1 - p_H) \beta I, \quad (2.7)$$

while welfare with zero net economic ownership is equal to

$$W_0 \equiv p_L (y_1 + y_2) + (1 - p_L) [1 - \delta (1 - \lambda)] y_2. \quad (2.8)$$

Since  $W_- > W_0$  for  $p_H$  sufficiently high, and  $W_- < W_0$  for  $p_H$  close to  $p_L$ , there exists a cutoff  $p_H^* > p_L$  such that for  $p_H > p_H^*$  it holds that  $W_- > W_0$ , and for  $p_H < p_H^*$  we have  $W_- < W_0$ . This result says that a policy to cap net economic ownership to be nonnegative can reduce welfare. This and other results derived in this section are summarized in Proposition 8.

**Proposition 8** *The following results hold regarding intervention and efficiency in CDS markets:*

- (1) *For continuation values below  $\underline{y}_2$  and above  $\overline{y}_2$ , overinsurance (restricting net economic ownership to be nonnegative) is inefficient (efficient) if and only if the project can be financed without overinsurance. The inefficiency (efficiency) of overinsurance is increasing (decreasing) in the projects' continuation value and decreasing in its probability of success.*
- (2) *For continuation values between  $\underline{y}_2$  and  $\overline{y}_2$ , there exists a cutoff  $p_H^* > p_L$  such that overinsurance (restricting net economic ownership to be nonnegative) is inefficient (efficient) if and only if  $p_H < p_H^*$ .*
- (3) *Just-insurance (restricting net economic ownership to be positive) is efficient (inefficient).*

Proposition 8 shows that for continuation values that are either sufficiently high or small enough, CDS markets can be inefficient if they lead to overinsurance and if projects can be

financed without CDSs. However, the inefficiency caused by the empty creditor problem is likely to be small in these cases. High continuation values are associated with high effort and high probability of success, which reduces the probability of default and liquidation.

For low continuation values, the inefficiency of empty creditors is reduced since forgone renegotiation proceeds under liquidation are small. Our results suggest that constraining the lender's net economic ownership to be nonnegative is unlikely to reduce the inefficiencies caused by the empty creditor problem. If along with these results one also considers that CDSs increase lenders' payoffs and debt capacity, then one might conclude that not allowing for negative net economic ownership can be harmful. In fact, when the probability of success is low ("busts"), overinsurance only occurs for borrowers with low continuation values (inefficiency due to empty creditors is small). Accordingly, policies constraining CDSs are not only unlikely to reduce the empty creditor problem, but also likely to reduce credit availability when firms need it the most.

For continuation values in the intermediary range, not allowing for CDS overinsurance can be inefficient whether or not projects can be financed by overinsured creditors. Since overinsurance minimizes the moral hazard problem and helps the implementation of high effort, it increases projects' payoffs. This is particularly true when agency problems are severe and the state of the economy is such that projects are likely to succeed ("booms"). Our model thus casts doubt on the benefits of capping CDS insurance.

The results of this section characterize the role CDSs play in borrower-creditor relations and their impact on the availability of credit. The analysis also discusses implications for optimal regulation. To sum up, although CDS overinsurance may cause the empty creditor

problem, our model shows that overinsurance is more likely to be observed when expected inefficiencies associated with empty creditors are lowest. In addition, we show that the efficient effort levels are generally induced along with overinsurance, further reducing the probability of default and inefficient liquidation. Our model implies that these effects have an impact on credit availability, suggesting that they need to be more fully appreciated by researchers in the field and policymakers.

## 2.7 Empirical Implications

We dedicate this section to the discussion of our model’s implications. We do so presenting a non-exhaustive list of testable empirical predictions, some of which are summarized in Table 2.1. We believe that examining these predictions would deepen our understanding of the CDS markets and their impact on corporate financing and economic efficiency.

*Implication #1: Lenders of firms that have higher probability of success and larger continuation values benefit the most from the existence of CDSs.*

This implication follows from lenders deriving greater benefits from CDS overinsurance when borrowers have higher probability of success (“safer firms”) and higher continuation values (“larger firms”). This implies that CDS are more likely to be written on firms that are safer and larger, and that these firms are likely to experience more favorable credit terms.

*Implication #2: The incidence of negative net economic ownership is increasing in firms’ probability of success.*

This implication follows from the fact that a higher probability of success implies that



lenders' payoff are weighted more towards outcomes associated with investment success. As a result, the extra debt repayment extracted when the project succeeds compensates for the forgone renegotiation proceeds when the project fails.

*Implication #3: The incidence of negative economic ownership is higher in booms and lower in busts; i.e., net economic ownership is countercyclical.*

This result is a corollary of Implication #2. During booms, the economy's overall probability of success is higher, reducing the opportunity cost of overinsurance. The opposite holds in periods of downturns.

*Implication #4: Among firms with CDSs written on their debt, the probability of bankruptcy given default is increasing in firms' probability of success.*

This implication follows from Implication #2 along with the following results: (1) negative net economic ownership leads to bankruptcy given default; and (2) zero negative net economic ownership leads to successful out-of-court renegotiation.

*Implication #5: After the emergence of CDS markets, firms' probability of bankruptcy given default should be higher in booms and lower in busts; i.e., the probability of bankruptcy given default is procyclical.*

This implication is a corollary of Implication #4 and suggests that CDS-led bankruptcy (out-of-court restructuring) probabilities given distress are higher (lower) in booms. The opposite holds for busts. Simply put, the empty creditor problem is procyclical.

*Implication #6: Among firms with CDSs written on their debt, the incidence of agency costs is smaller for firms with higher probability of success.*

This implication follows from Implication #2 along with the fact that negative net economic ownership minimizes managers' payoffs under default. The latter implies that managers have higher incentives to avoid default by implementing high effort, hence increasing the probability of success.

While our model's predictions are new and have not been directly taken to the data, some reported empirical regularities are consistent with our theory. We argue, for example, that CDSs are more beneficial for firms that are safer and have higher continuation values. This result is interesting and stands in contrast to common intuition that riskier firms would benefit the most from the existence of CDS markets. Consistent with our theory, however, recent studies by Ashcraft and Santos (2009) and Hirtle (2009) find that safer and larger firms have benefited the most from CDS contracts (for example, by paying lower spreads on their bank loans). Song and Uzmanoglu (2010) explore the financial crisis and find evidence that firms associated with safer banks (typically safer firms) observed smaller CDS spreads in the 2008–9 period.

Another unintuitive prediction of our model is the procyclicality of the empty creditor problem. This implies that the conditional probability of CDS-led bankruptcy given that a firm is in distress is higher in booms. Interestingly, starting with the prior that CDSs aggravate the empty credit problems in busts — the opposite of our model's prediction — Bedendo et al. (2010) fail to find evidence that the CDS contracting leads to a higher incidence of bankruptcies (relative to out-of-court restructurings) during the financial crisis (see also Mengle (2009) and Aspel and Iden (2010)).

A number of other predictions listed above can be directly taken to the data. Empirical research on CDS is still in its infancy and this strikes us as setting in which models describing rich sets of creditor–borrower relations are particularly useful in guiding empirical work.

## 2.8 Concluding Remarks

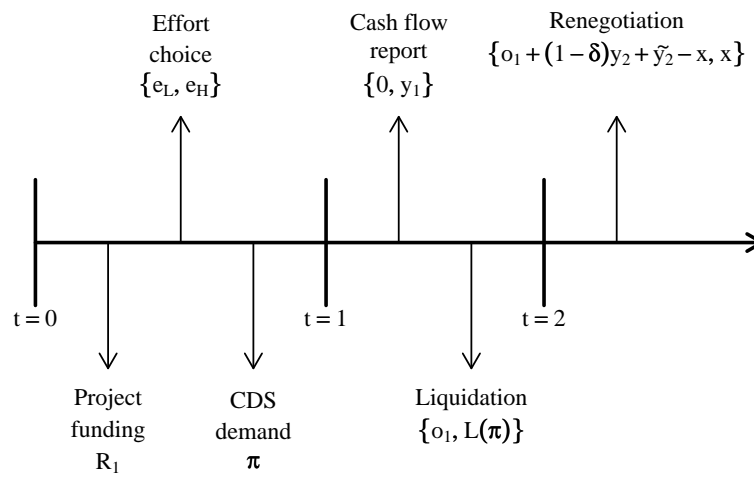
Financial innovation is the hallmark of capital markets in developed economies. At the same time, financial innovations have also preceded virtually all major economic crises in history (see Kindleberger (2000)). The 2008–9 crisis has brought renewed interest in innovation and regulation of financial markets. A great deal of attention, in particular, has been given to CDS contracts as these derivatives seemed to play a role in the demise of numerous banks and industrial firms during the crisis. Examples range from Lehman Brothers to GM and Six Flags. We argue that while we might have observed a high degree of association between bankruptcies and CDS contracts during the crisis, it is hard to conclude that CDSs led to excessive, inefficient liquidation in that period. To draw that inference, one needs a benchmark model describing the connections between CDS contracting and the economy.

We develop a model of optimal CDS contracting when investment is subject to moral hazard and wealth verification is imperfect. To our knowledge, this model is the first to show how lenders choose between debt payments and restructuring proceeds — accounting for the state of the economy — when selecting the optimal amount of CDS protection. Counter to previous arguments in the literature, we show that CDS overinsurance is more likely to occur in booms, when it boosts firm debt capacity and increases the number of projects with positive NPV that receive funding. CDS contracts alleviate credit rationing during reces-

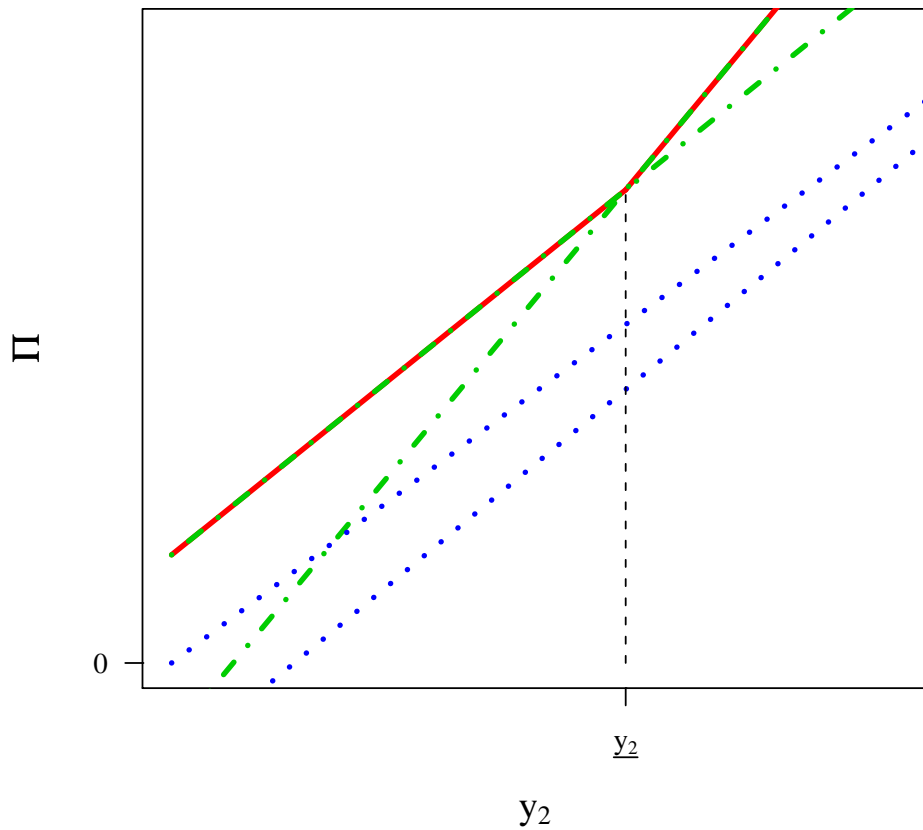
sions, but in those times CDS overinsurance may prompt the liquidation of firms with less promising prospects (firms that would likely be rationed in the absence of CDS). Our model demonstrates that the empty creditor problem is procyclical. Moreover, it implies that the casual observation that CDS contracts are associated with bankruptcies in the crisis does not imply that those contracts harm financial efficiency.

A number of recent proposals aim at constraining the amount and ownership of CDS contracts on a firm's debt. Most notably, they suggest that the degree CDS-based insurance should be lower than — or, at a maximum, equal to — the amount of economic exposure of lenders. Our paper cautions about the potential effects of these proposals on the availability of credit and on financing efficiency. Complex contracts such as CDSs are inexorably linked to the forms of financing arrangements we will be seeing in future years, as financial markets become more sophisticated and integrated. One has to be careful about imposing constraints on the markets for these contracts at an early stage of their development. At the same time, one needs to better understand how these contracts work and the types of inefficiencies they address. In this way, one might be able to more fully benefit from what these contracts have to offer.

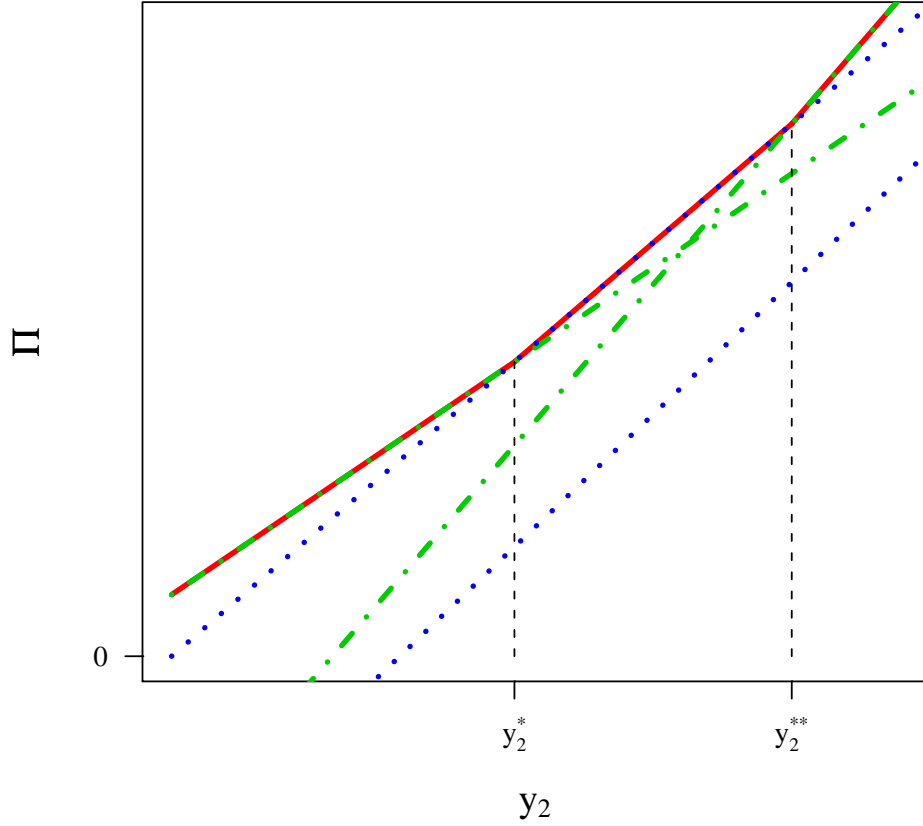
## 2.9 Figures and Tables



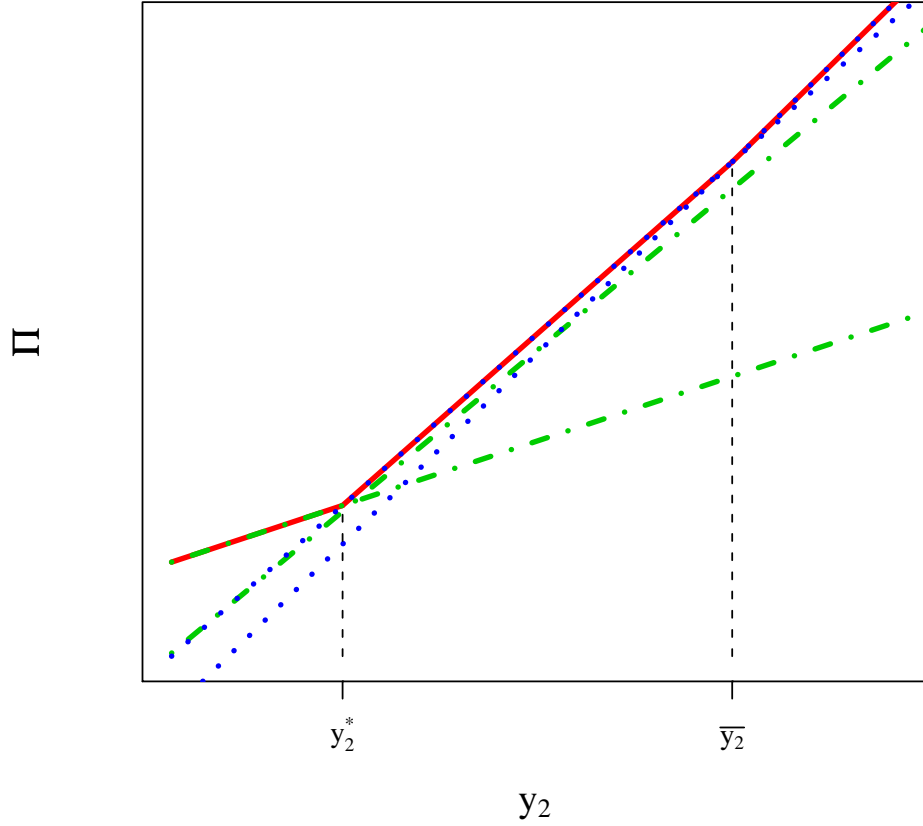
**Figure 2.1:** Timing of the Game



**Figure 2.2:** Lender Overinsures. The horizontal axis represents the project's continuation value,  $y_2$ . The vertical axis represents the lender's payoff,  $\Pi$ . The two dotted lines represent the lender's payoffs if she chooses to have zero net economic ownership. The two dashed-dotted lines denote the lender's payoffs if he overinsures. The solid (overimposed) lines describe the lender's optimal payoff schedule. The lender's choice of debt repayment induces low effort for continuation values below  $\underline{y}_2$  and induces high effort for values above that cutoff.



**Figure 2.3:** Lender Not Always Overinsure for High Continuation Values. The horizontal axis represents the project's continuation value,  $y_2$ . The vertical axis represents the lender's payoff,  $\Pi$ . The two dotted lines represent the lender's payoffs if she chooses to have zero net economic ownership. The two dashed-dotted lines denote the lender's payoffs if he overinsures. The solid (overimposed) lines describe the lender's optimal payoff schedule. The lender's choice of debt repayment induces low effort for continuation values below  $y_2^{**}$  and induces high effort for values above that cutoff.



**Figure 2.4:** Lender Does Not Overinsure for High Continuation Values. The horizontal axis represents the project's continuation value,  $y_2$ . The vertical axis represents the lender's payoff,  $\Pi$ . The two dotted lines represent the lender's payoffs if she chooses to have zero net economic ownership. The two dashed-dotted lines denote the lender's payoffs if he overinsures. The solid (overimposed) lines describe the lender's optimal payoff schedule. The lender's choice of debt repayment induces low effort for continuation values below  $\bar{y}_2$  and induces high effort for values above that cutoff.



**Table 2.1:** CDS–Effort Equilibrium Outcomes

The table entries represent the equilibrium levels of CDS insurance (overinsurance vs. no-overinsurance) and effort level (low vs. high) for different combinations of investment success probability (across rows) and continuation values (across columns). O, NO, LE, and HE denote, respectively, overinsurance, no-overinsurance, low effort, and high effort.

Probability of Investment Success	Continuation Value					
	$< y_2^*$	$< \underline{y}_2$	$< y_2^{**}$	$< \overline{y}_2$	$< y_2^{***}$	$> y_2^{***}$
$p_H > p_L \geq c(p_H) > c(p_L)$	O–LE	O–LE	O–HE	O–HE	O–HE	O–HE
$p_H \geq c(p_H) > c(p_L) > p_L$	O–LE	NO–LE	NO–LE	O–HE	O–HE	O–HE
$c(p_H) > p_H > c(p_L) > p_L$	O–LE	NO–LE	NO–LE	O–HE	O–HE	NO–HE
$c(p_H) > c(p_L) \geq p_H > p_L$	O–LE	NO–LE	NO–LE	NO–LE	NO–HE	NO–HE

# Chapter 3

## Optimal Financing with CDS Markets

### 3.1 Introduction

The credit default swap (CDS) market has experienced explosive growth since 2003. According to the International Swaps and Derivatives Association (ISDA), the outstanding amount of CDS contracts was under \$4 trillion in 2003 and reached a peak of \$62 trillion in 2007. The 2008–9 crisis brought scrutiny to CDSs. Many have identified CDSs as a main culprit for the crisis and new regulatory legislations have been enacted to curb CDS markets.<sup>1</sup> Surprisingly, however, little is known about CDSs. We know little about the role of CDSs in financial markets, what contracting inefficiencies they address, whether they affect the supply of credit, and how economic conditions impact the demand for CDS.

One can argue that CDSs alleviate the frictions of capital markets by improving hedging opportunities and allowing for greater credit supply and better terms for firms. Accordingly,

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<sup>1</sup>In his testimony before congress on September 23, 2008, the SEC chairman Christopher Cox has called for regulatory scrutiny of the CDS market. George Soros said “Some derivatives ought not to be allowed to be traded at all. I have in mind credit default swaps. The more I’ve heard about them, the more I’ve realised they’re truly toxic” (see “UPDATE 1-Ban CDS as “instruments of destruction” - Soros”, Reuters, June 12, 2009). Signed by President Barack Obama in 2010, the Dodd-Frank Act (HR #4173) gives the SEC regulatory authority over swaps, including CDSs. The new law requires the reporting of trades, sets position limits, imposes margin requirements, and moves swaps away from over-the-counter markets into organized exchanges.

one would expect riskier firms to benefit the most from the existence of CDS insurance. In contrast, recent evidence by Ashcraft and Santos (2009) and Hirtle (2009) find that CDSs are more liquid for safer, larger firms and that these firms have benefited the most from CDS markets (for example, by paying lower interest on their bank loans once CDSs are written on their bonds). This puzzling result suggests that CDSs can be beneficial to some firms and less so to others. Understanding how this arises in equilibrium should be seen as fundamental for a better understanding of CDS markets and for the design of welfare improving policies following the financial crisis.

A CDS is a bilateral agreement between a debt protection seller and a debt protection buyer. The buyer makes periodic payments to the seller in exchange for compensation when there is a credit event — a borrower defaults on the debt it issued. In the real world, lenders have differentiated exposures to borrowers' risk of distress (Minton, Stulz and Williamson (2009) and Knaup and Wagner (2009)) and hence heterogeneous incentives to buy CDS protection. One would think that borrowers would get better financing terms with those lenders who benefit the most from improved hedging mechanisms. However, as pointed out by Hu and Black (2008a,b), lenders protected by CDS might have low incentives to participate in out-of-court restructurings of distressed firms since formal default triggers immediate compensation for their exposure.

While there is a growing literature examining the impact of CDS on lender-borrower relationships, the literature lacks a theoretical framework to answer some of the most fundamental questions about CDSs. How the existence of lenders that have differentiated benefits from CDSs affects firms' policies? Which firms benefit the most from the existence of CDS?

Answering these questions allows for the characterization of welfare and financing policies as functions of lenders and borrowers' attributes. These, in turn, can be used to enhance the regulatory architecture.

This paper addresses these questions by developing a model of optimal financing policies in the presence of CDSs when competitive lenders have different exposures to borrowers' distress risk. To my knowledge, this paper is the first to consider how heterogeneous lenders affect firms' probability of default given distress and financing policy. This study also shows why *safer* firms — those with lower probability of being in distress — have a *higher* probability of having CDSs written on their debt and experience better financing terms. The model I introduce is useful in allowing us to understand important facets of the CDS markets, which until now were not fully understood. I show that a simple tax scheme based on fixed fees that are proportional to lenders' exposure and increasing in the borrower's risk of distress can make CDSs beneficial to those firms that would be harmed otherwise. I also demonstrate that if lenders' exposures to the borrower's risk of distress are unknown to the borrower, then the beneficial properties of CDSs are reduced.

The model I develop also sheds light on recent empirical studies on CDSs. Ashcraft and Santos (2009) do not find evidence that the average firm experiences improved lending terms. They argue that one possible explanation is lack of liquidity in the CDS market. The authors in turn find evidence that firms for which CDSs are more liquid benefit more from CDSs. My model is consistent with this result, but suggests the link is non-causal as liquidity is an endogenous variable: safer firms benefit more from CDS and lean more heavily on lenders that profit the most from CDS insurance, which are exactly those that demand CDSs more often.

Let me provide some details of the model. A borrower must get funds from heterogeneous and competitive lenders to finance a project. These lenders have different exposures to the borrower's risk of distress, which take the form of additional losses triggered in the event the borrower is in distress. This captures the idea that defaulting loans can cause lenders to be in distress. These losses depend negatively on the amount of resources available to lenders and the dependence is more sensitive for lenders with higher risk exposure. Risk exposures could result from the concentration of lenders' portfolio on loans with similar risk as that of the borrower, or from lenders' absolute sensitivity to credit risk — which depends on factors such as size of portfolio and leverage. As a consequence, CDS protection might be more useful to some lenders than to others.

If the borrower is in distress, insured lenders have stronger preferences for default as this triggers CDS payment and reduces their losses. The risk faced by these lenders is that the fraction of lenders that buy CDS may be insufficiently large to cause the borrower to default. In this case, the borrower can renegotiate the debt in the hands of uninsured lenders and fully repay protected lenders. As a result, the borrower successfully renegotiates his debt out-of-court, avoiding a credit event. This outcome can be especially harmful to overinsured lenders — lenders that buy an amount of protection beyond the face value of debt. The risk of successful private workouts is justified since standard CDS contracts, as defined by the International Swaps and Derivatives Association, do not recognize out-of-court restructuring as a credit event. Although it is possible to include restructuring as a credit event in a CDS contract, CDS payments are triggered only if the restructuring binds all debt holders.<sup>2</sup>

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<sup>2</sup>Notably, a corporate private workout has never triggered a credit event in the U.S. (Bedendo et al (2010)).

I model this coordination game among lenders as a global game and establish unique conditions for liquidation.<sup>3</sup> Lenders coordinate in buying CDS protection whenever the fundamental regarding the borrower's cash flow in distress exceeds a cutoff. The global game structure permits the direct computation of the liquidation threshold. This threshold depends on the cost-benefit ratios of insuring for lenders of different types and on the proportion of each type in the total amount of debt outstanding. The higher the fraction of debt in the hands of a group of lenders, the more influential this group is in determining the liquidation threshold. Lenders with higher exposure (low cost-benefit ratios) contribute to a lower cutoff, while lenders with lower exposure (higher cost-benefit ratios) contribute to a higher cutoff.<sup>4</sup> The reason is that, the higher the exposure, the less is the optimism about the occurrence of a credit event necessary to make a lender indifferent between insuring and not insuring. Finally, the lower is the debt repayment of a particular group, the higher is the cost of coordination failure and the higher is this group's cost benefit ratio.

When choosing its financing policy, a borrower faces the following tradeoff. Lenders that are highly exposed to borrowers' risk benefit the most from CDS insurance, allowing the borrower to finance his debt with lower repayments. Increasing the proportion of these lenders reduces the overall face value of debt but decreases the liquidation cutoff. Therefore, safer borrowers finance more heavily with more exposed lenders and enjoy lower debt repayments even though this results in a lower liquidation cutoff and in a higher probability of liquidation given distress. Riskier borrowers are relatively more penalized by a lower liquidation

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<sup>3</sup>For a summary of global game results and the most commonly used setups see Morris and Shin (2003).

<sup>4</sup>This is consistent with recent evidence provided by Song and Uzmanoglu (2010). Their paper explores the financial crisis and finds evidence that firms associated with safer banks observed smaller CDS spreads in the 2008–9 period.

threshold and therefore optimally choose to lean more heavily on less exposed lenders. Since lenders' payoffs increase in the probability of a credit event, a borrower can reduce the repayment to a particular group of lenders by increasing that of others. Hence, the safer the borrower, the higher the proportion of more exposed lenders and the higher the spread — repayment of riskier lenders minus that of safer lenders. Overall, firms that are sufficiently safe benefit from CDSs, as the surplus brought about in terms of reduced repayments in good times surpasses the extra distress costs. At the other end of the spectrum, firms with sufficiently high distress probabilities have inefficiently low liquidation cutoffs. These firms are frequently liquidated once in distress, so that CDSs are harmful to them.

After identifying the sources of inefficiency in CDS markets, I look for policies that might improve welfare. In that vein, I first examine policies that would make risky borrowers benefit from CDS markets. I show that a simple scheme in which lenders pay a fixed fee in order to buy CDS protection can reduce inefficient liquidation. The fee is higher for CDSs written on the debt of riskier borrowers and for lenders that are more exposed to borrowers' risk of distress. As a result, it becomes harder for lenders to coordinate, which raises the liquidation cutoff. This fee system does not require information regarding lenders' private information and their distributions or the proportion of each type of lender holding the borrower's debt. It only requires a measure of the exposure of each group of lenders to borrower's distress and the probability of distress, where the former can be obtained from balance sheet data and the latter from CDS spreads.

I compare this fee scheme with commonly proposed interventions, namely, limiting voting in restructuring decisions to lenders with positive net economic ownerships — CDS protec-

tion below the amount of debt owed by the firm (Hu and Black (2008a,b)), making voluntary renegotiations trigger a credit event, and capping the amount of protection to the amount of debt owed by the borrower (Spamann (2010) and Bolton and Oehmke (2011)).<sup>5</sup> I show that the fee system of this paper is superior to those proposals if the borrower’s probability of distress is sufficiently high — exactly the range for which CDSs can be harmful otherwise. The reason is that these proposals work as a coordination tool to lenders, resulting in an excessively low liquidation cutoff and a high distress cost.

Things get more complicated if lenders’ exposures are unknown to the borrower at the time of financing. This happens when lenders’ distress risk is hard to measure objectively. In this case, lenders that are more exposed want to look like they have lower exposure in order to enjoy higher repayments.<sup>6</sup> This would harm specially safer borrowers that lean more heavily on these types of lenders. Therefore, borrowers need to inefficiently increase the proportion of safer lenders, reducing the surplus brought about by CDSs. This result provides some support for proposals that aim at making information regarding lenders’ risk exposure more transparent and accessible. It also suggests that finding better measures and indicators of lenders’ risk exposure can greatly improve market efficiency.

My model is related to an emerging literature on “empty voting.” This literature emphasizes the ability of informed agents to make gains by inefficiently influencing voting outcomes. Brav and Matthews (2011) develop a model in which a proposal needs shareholder approval.

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<sup>5</sup>See “Restructuring credit event may be reviewed - ISDA,” Reuters, August 3, 2011 for considerations on extending restructuring credit event definitions.

<sup>6</sup>There is large evidence in the banking literature that banks manipulate reports on their loan loss data (see Wall and Koch (2000) and Hasan and Wall (2004)). There is also evidence of delaying loan provisioning (Laeven and Majnoni (2003)) and overstating the value of distressed assets (Huizinga and Laeven (2009)).



Traders with superior information know the true value of the proposal and combine negative net economic ownership and high voting power by buying and selling shares. Traders may then approve (and profit from) proposals that are detrimental to firm welfare. Spamann (2010) focuses on how traders build up empty voting positions via derivatives instruments. In his model, hedge funds derive gains from their positions because they are better informed than other agents about derivatives and securities markets. These models have two important features. First, they assume that one agent is systematically better informed than the others. Second, while informed agents vote strategically, the votes of other market participants are assumed to be given by a random variable that is exogenous to the model. The present paper focus on implications of strategic uncertainty — players’ uncertainty about the actions of other players — rather than heterogenous uncertainty regarding a fundamental (e.g., the value of a proposal). Importantly, the liquidation outcome in my model is endogenously determined by all participants acting strategically.

There is also a growing literature on the link between CDSs and creditor–borrower relations. Most studies in this literature focus on the impact of CDSs on adverse selection and moral hazard problems. Duffee and Zhou (2001) show that CDSs can alleviate “lemons problems” in credit risk-transfer markets. Thompson (2007) extends Duffee and Zhou by allowing asymmetric information in the credit insurance market. He derives welfare properties when loan sales and CDSs emerge as banks’ optimal risk-transfer instruments. In a similar vein, Parlour and Winton (2008) analyze when loan sales and CDSs emerge in equilibrium and characterize efficiency in terms of risk transferring and bank monitoring. Parlour and Plantin (2008) further investigate the effect of CDS markets on banks’ incentive to monitor

(see also Morrison (2005)). Arping (2004) studies CDS contracting in the presence of moral hazard. He shows that CDSs increase the commitment of lenders to terminate projects if borrowers misbehave. Bolton and Oehmke (2011) study CDSs in a model where borrowers have no commitment to repay their debt (cash flows are non-verifiable). CDSs increase lenders' bargaining power, working as a commitment device that increases borrowers' debt capacity. The authors show that the drawback of CDSs is that lenders often overinsure, causing the empty creditor problem. Campello and Matta (2011) show that the empty creditor problem might not be so severe as it depends on the probability of investment success. Overinsurance is higher when the probability of investment success is higher or the probability of distress is lower. None of these papers consider the impact of heterogeneous lenders on the probability of liquidation and its implications for how borrowers choose which lenders to borrow from.

The remainder of the paper is organized as follows. Section 2 describes the model setup. Section 3 examines the equilibrium of the CDS coordination game and its implications for the probability of default given distress. Section 4 derives the borrower's optimal financing policy with and without CDSs and derive comparative statics. Section 5 extends the model to allow for uncertainty regarding lenders' risk exposure and the investigation of public policies. Section 6 presents a set of empirical implications. Section 7 concludes the paper. All proofs are in Appendix D.

## 3.2 The Model

The economy lasts for three periods  $t = 0, 1, 2$  and is populated by a borrower, a continuum of lenders indexed by  $i \in [0, 1]$ , and a continuum of atomistic CDS providers. All agents are

risk neutral and there is no discounting. The borrower is penniless and needs an amount  $\alpha \in (0, 1)$  of funds to finance his project in  $t = 0$ . The borrower can obtain finance from lenders, each of whom is endowed with one unit of funds in  $t = 0$  and one unit of an illiquid and indivisible asset that generates a value of  $a$  in  $t = 1$ . Lenders can be from two different groups indexed by  $g \in \{R, S\}$ . The proportion of lenders from group  $R$  is  $\beta \in (0, 1)$  (and that from group  $S$  is  $1 - \beta$ ). It is possible for the borrower to finance his project only with lenders of a particular group, i.e.,  $\alpha \leq \min \{\beta, 1 - \beta\}$ .<sup>7</sup>

If the project is financed it generates cash flow  $c \in \{y(\theta), \bar{y}\}$  in  $t = 2$ . With probability  $1 - \lambda$  the project succeeds and  $c = \bar{y}$  — assumed to be large — and with probability  $\lambda$  the project fails and  $c = y(\theta)$ . When the project's cash flow is  $\bar{y}$ , the borrower is “sound”, while when the cash flow is  $y(\theta)$ , the borrower is “in distress”. I assume that  $c$  is observable, but non-verifiable. Cash flow  $y(\theta)$ , which depends on the fundamental  $\theta$ , is given by  $y(\theta) = \max \{0, \theta\}$ , where  $\theta$  is distributed according to a continuously differentiable and strictly positive density  $h(\cdot)$  with support on  $\mathbb{R}$ . The true value of  $\theta$  is unknown to all participants until  $t = 2$ .

Each lender  $i$  receives a private noisy signal in  $t = 1$  given by

$$x_i = \theta + \sigma \eta_i, \tag{3.1}$$

where the noise terms are i.i.d. with continuous density  $f_{g_i}(\cdot)$  with support on  $\mathbb{R}$ .<sup>8</sup>

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<sup>7</sup>Initially, lenders' groups are assumed to be common knowledge. Later, the model is extended to accommodate the possibility that lenders' groups are unknown to the borrower.

<sup>8</sup>One could argue it would be more realistic to assume that the borrower is better informed than lenders about the probability of distress. Adding this assumption would add more complexity without causing any qualitative change to my results. The reason is that lenders' signals concern the project's cash flow given distress.

In  $t = 0$ , the borrower offers financing contracts to lenders. A financing contract is represented by  $(p_g, r_g)$ , where  $p_g$  is the probability that a lender from group  $g$  finances the project and  $r_g$  is the repayment to the lender.

Given its signal about  $\theta$ , each lender  $i$  decides the amount of credit insurance  $s_i$  to buy from CDS providers in  $t = 1$ . CDS providers act as price takers and decide whether to offer a contract that pays 1 unit if there is a credit event in  $t = 2$ , in exchange for a fee of  $f$  (paid up front). A credit event occurs if the borrower defaults on his debt — if the borrower fails to repay his debt in full — and if out-of-court restructuring is unsuccessful. Default and renegotiation are described next.

Similar to Bolton and Jeanne (2005, 2007), I assume that the borrower has incentives to repay his debt only as a way of avoiding default costs. I assume that failure to repay the debt in full when the realized cash flow is  $\bar{y}$  results in a “reputational cost” given by  $\gamma\bar{y}$ , which I assume equals  $\bar{y}$  (i.e.,  $\gamma = 1$ ) to streamline the analysis. In other words, the borrower loses all the output  $\bar{y}$  if he is able to repay all his debt but chooses to default instead.

If the borrower is in distress, i.e.,  $c = y(\theta)$ , then no reputational cost applies. However, I assume that there is a cost  $\rho y(\theta)$  if the borrower cannot successfully renegotiate his debt out-of-court. This assumption is supported by evidence that out-of-court restructurings are considerably less costly than bankruptcies to debtor firms (e.g., Gilson et al (1990)). This cost, which for simplicity I assume equals  $y(\theta)$  (i.e.,  $\rho = 1$ ), can be interpreted as a sanction imposed by the investors that financed the borrower. Renegotiation proceeds as follows.

In  $t = 2$ , each lender’s CDS position can be observed by the borrower and the other lenders but cannot be verified by a court. Upon the realization of  $y(\theta)$ , the borrower offers

$q_i$  to lender  $i$ . If  $q_i < r_{g_i}$ , then lender  $i$  receives  $q_i$  if he accepts and his outside option  $o_i \in \{0, s_i\}$  if he rejects. If a lender rejects the borrower's offer, then renegotiation fails and each lender receives his outside option.

[Figure 3.1 about here]

Lenders are heterogenous with respect to their payoffs. Let  $v_i$  be lender  $i$ 's payoff if renegotiation succeeds. Lender  $i$ 's incremental payoff of insuring over not insuring if he finances the borrower is

$$\pi_i(s_i, P, \theta) = \begin{cases} -fs_i + \lambda(s_i + L_{g_i}(a) - L_{g_i}(s_i + a - fs_i)) & \text{if renegotiation fails} \\ -fs_i + \lambda(v_i + L_{g_i}(a) - L_{g_i}(v_i + a - fs_i)) & \text{if renegotiation succeeds} \end{cases} \quad (3.2)$$

The function  $L_{g_i}(I_i)$  represents the additional loss suffered by lender  $i$  if the borrower is in distress, where  $I_i$  is lender  $i$ 's the final income and  $L_R(a) = L_S(a) = L(a)$ . This loss captures lenders' exposure to the borrower's risk of distress and is assumed to decrease in the final amount of funds, i.e.,  $L'_{g_i}(I_i) < 0$ .<sup>9</sup>

I assume lenders of group  $R$ , which I refer to as “risky” lenders, have a higher exposure to the borrower's risk of distress. Therefore, CDS insurance is more attractive to these lenders than to those from group  $S$ , which I refer to as “safe” lenders. Formally,  $L_g(\cdot)$  is assumed to be such that  $L'_R(I) < L'_S(I)$  for all  $I$ . One can think of the portfolios of lenders of group  $R$  as containing mostly loans with default risk similar to that of the borrower. Or it could be that these lenders have higher absolute sensitivity to credit risk — which depends on factors

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<sup>9</sup>This way of modelling lenders' exposure to distress follows Duffee and Zhou (2001). It is a shortcut to capture this dependence without having to model lenders' capital structure and default conditions.

such as size of portfolio and leverage. Therefore, if the borrower is in distress, these lenders face a higher probability of being in distress as well — facing extra liquidity needs and harder access to capital markets. Lenders of group  $S$  are safer in that they are less exposed to the borrower's risk of distress, which translates into a lower sensitivity to the amount of available funds. These lenders can be seen as having more diversified loan portfolios such that their soundness are less affected in the event the borrower is in distress.<sup>10</sup>

Finally, if  $V$  is the borrower's proceeds if renegotiation succeeds, then his payoff is

$$u_b(P, \theta) = \begin{cases} (1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) & \text{if renegotiation fails} \\ (1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) + \lambda V & \text{if renegotiation succeeds} \end{cases} \quad (3.3)$$

### 3.3 The CDS Coordination Game

I examine the coordination problem among lenders regarding their decisions to buy CDS protection. I first argue that lenders' decision to insure depends only on the percentage of the total debt outstanding for which there is CDSs written on rather than on the aggregate amount of CDS protection. As a consequence, the coordination game is simplified and lenders need only worry about other lenders' decision to insure. Then I show that the each group equilibrium strategies are in the form of switching strategies around the group's cutoff regarding the borrower's economic fundamental. Finally, since I want to focus more on strategic uncertainty rather than fundamental uncertainty, I make lenders' signals about the fundamental be nearly precise and derive the unique liquidation cutoff that lenders agree upon.

Assuming a lender accepts the offer if he is indifferent, the borrower offers  $q_i = 0$  to a

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<sup>10</sup>For evidence of variation in lenders' exposure to loans of different risks see Knaup and Wagner (2009).

lender with  $o_i = 0$ . It is also clear that the borrower offers  $\min\{r_{g_i}, s_i\}$  to a lender with  $o_i = s_i$ . Let  $\bar{r} \equiv p_R \beta r_R + p_S (1 - \beta) r_S$ . Renegotiation fails (a credit event occurs) if

$$\frac{y(\theta)}{\bar{r}} < P \equiv \frac{1}{\bar{r}} \int_{\{i: o_i = s_i\}} \min\{r_{g_i}, s_i\} di, \quad (3.4)$$

and succeeds (a credit event does not occur) otherwise.

Therefore, lender  $i$ 's incremental payoff of insuring over not insuring if he finances the borrower is

$$\pi_i(s_i, P, \theta) = \begin{cases} -fs_i + \lambda(s_i + L_{g_i}(a) - L_{g_i}(s_i + a - fs_i)) & \text{if } \frac{y(\theta)}{\bar{r}} < P \\ -fs_i + \lambda(\min\{r_{g_i}, s_i\} + L_{g_i}(a) - L_{g_i}(\min\{r_{g_i}, s_i\} + a - fs_i)) & \text{if } \frac{y(\theta)}{\bar{r}} \geq P \end{cases}. \quad (3.5)$$

Since  $\pi_i(s_i, P, \theta, f)$  is strictly increasing in  $s_i$  for  $s_i < r_{g_i}$ , lender  $i$  insures only if he buys CDS protection in an amount at least as great as his repayment, i.e.,  $s_i \geq r_{g_i}$ . Let  $l_g$  be the fraction of lenders of group  $g$  that buy CDS. The borrower defaults if

$$\frac{y(\theta)}{\bar{r}} < P = \frac{1}{\bar{r}} [l_R \beta r_R + l_S p_S (1 - \beta) r_S], \quad (3.6)$$

and does not default if otherwise. Condition 3.6 states that the borrower's cash flow under distress is smaller than the total amount of outstanding debt that is insured.

One can rewrite lender  $i$ 's incremental payoff of insuring over not insuring as:

$$\pi_i(P, \theta) = \begin{cases} -fs_i + \lambda(s_i + L(a) - L_{g_i}(s_i + a - fs_i)) & \text{if } P > \frac{\theta}{\bar{r}} \\ -fs_i + \lambda(r_{g_i} + L(a) - L_{g_i}(r_{g_i} + a - fs_i)) & \text{if } P \leq \frac{\theta}{\bar{r}} \end{cases}. \quad (3.7)$$

Note that the payoff of lender  $i$  depends on his own amount of CDS insurance  $s_i$ , the borrower's fundamental  $\theta$ , and the fraction of lenders that insure. Lender  $i$ 's payoff does

not depend on the amount of CDS insurance chosen by the other lenders. This implies that lenders of group  $g$  should choose the same amount of insurance  $s_g$  in equilibrium. It also implies that the coordination problem among lender is regarding their decisions to either insure or not insure. Lenders' equilibrium strategies is characterized in the following proposition.

**Proposition 1** *The unique equilibrium among lenders after observing CDS fee  $f$  in  $t = 1$  has each lender  $i$  following a threshold strategy: insuring if  $x_i < x_{g_i}^*$  and not insuring if  $x_i \geq x_{g_i}^*$ .*

Proposition 1 states that lenders of a particular group insure if their signal regarding the borrower's cash flow in distress is below the group's threshold. It is important to note that when the true value of the fundamental lies between  $x_R$  and  $x_S$  there is miscoordination between riskier and safer lenders. This is mainly due to the fact that their signals are noisy.

Since my focus is on the strategic uncertainty among lenders rather than fundamental uncertainty (uncertainty regarding  $\theta$ ), I derive the equilibrium when  $\sigma \rightarrow 0$ , i.e., the uncertainty regarding the fundamental  $\theta$  becomes small.<sup>11</sup> Proposition 2 states that, in this case, lenders of each group agree upon a common liquidation cutoff.

**Proposition 2** *Let  $1 - \phi \equiv \frac{\theta}{\tau}$ . As  $\sigma \rightarrow 0$ , the thresholds  $x_R^*$  and  $x_S^*$  converge to a common critical fundamental  $\theta^*$  and lenders coordinate on insuring whenever  $\phi > \phi^*$  and not insuring whenever  $\phi < \phi^*$ , where*

$$\phi^* = \frac{p_R \beta}{\alpha} \frac{f s_R + \lambda [L_R(r_R + a - f s_R) - L(a) - r_R]}{\lambda [s_R + L_R(r_R + a - f s_R) - L_R(s_R + a - f s_R) - r_R]} + \frac{p_S (1 - \beta)}{\alpha} \frac{f s_S + \lambda [L_S(r_S + a - f s_S) - L(a) - r_S]}{\lambda [s_S + L_S(r_S + a - f s_S) - L_S(s_S + a - f s_S) - r_S]}.$$

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<sup>11</sup>One could imagine a setup in which lenders receive signals about what they will learn at the interim stage — and therefore lenders' decision either to buy or not CDS protection. This setup will have similar strategic effects since the important part is the feedback to the borrower's initial problem. The assumption that lenders' uncertainty vanish is a short cut for ease of analysis.



The threshold of Proposition 2 is computed as follows. Although the threshold lenders of each group have different beliefs regarding the probability that the fraction of insured debt is less than a certain number, the average of this beliefs is the uniform belief on the unit interval (see Steiner and Sakovics (2010)). Therefore, the average belief of a credit event is:

$$\frac{p_R \beta}{\alpha} (1 - \Pr(P \leq 1 - \phi^* | R)) + \frac{p_S (1 - \beta)}{\alpha} (1 - \Pr(P \leq 1 - \phi^* | S)) = 1 - (1 - \phi^*) = \phi^* \quad (3.8)$$

One can rewrite lenders' incremental payoffs using

$$\begin{cases} B_{g_i} \equiv \lambda(s_i - r_{g_i} + L_{g_i}(r_{g_i} + a - f s_{g_i}) - L_{g_i}(s_{g_i} + a - f s_{g_i})) \\ C_{g_i} \equiv f s_{g_i} - \lambda(r_{g_i} + L(a) - L_{g_i}(r_{g_i} + a - f s_{g_i})) \end{cases}$$

such that

$$\pi_i(s_i, P, \theta, f) = \begin{cases} B_{g_i} - C_{g_i} & \text{if } P > \frac{\theta}{\bar{r}} \\ -C_{g_i} & \text{if } P \leq \frac{\theta}{\bar{r}} \end{cases}. \quad (3.9)$$

Since the threshold lender of each group must be indifferent between insuring and not insuring then, it must be that:

$$\begin{aligned} (1 - \Pr(P \leq 1 - \phi^* | g)) (B_g - C_g) - \Pr(P \leq 1 - \phi^* | g) C_g &= 0 \\ \Rightarrow (1 - \Pr(P \leq 1 - \phi^* | g)) &= \frac{C_g}{B_g}. \end{aligned} \quad (3.10)$$

Therefore, 3.8 and 3.10 yield the cutoff presented in Proposition 2.

It is now possible to calculate, given  $f$ , lender  $i$ 's incremental payoff of financing the

borrower over non-financing:

$$U_{g_i}(r_{g_i}, s_{g_i}) = H(\bar{r}(1 - \phi^*)) \{ (1 - \lambda)(r_{g_i} + a - fs_{g_i}) + \lambda[s_{g_i} + a - fs_{g_i} - L_{g_i}(s_{g_i} + a - fs_{g_i})] \} + (1 - H(\bar{r}(1 - \phi^*))) \{ (1 - \lambda)(r_{g_i} + a) + \lambda(a - L(a)) \} - (1 + a).$$

Because the CDS market is competitive, it must be that  $f = \lambda$ . Since  $U_{g_i}(r_{g_i}, s_{g_i})$  is strictly increasing in  $s_i$ , it is optimal for every lender  $i$  to choose  $s_i = \frac{a}{\lambda}$ . Therefore, lenders' payoffs and the liquidation cutoff reduce to

$$(1 - \lambda)r_g - \lambda \left[ H(\bar{r}(1 - \phi^*)) L_g\left(\frac{a}{\lambda}\right) + (1 - H(\bar{r}(1 - \phi^*))) L(a) \right] - 1 \quad (3.11)$$

and

$$\phi^* = \frac{p_R \beta}{\alpha} \frac{a + \lambda [L_R(r_R) - L(a) - r_R]}{a + \lambda [L_R(r_R) - L_R(\frac{a}{\lambda}) - r_R]} + \frac{p_S (1 - \beta)}{\alpha} \frac{a + \lambda [L_S(r_S) - L(a) - r_S]}{a + \lambda [L_S(r_S) - L_S(\frac{a}{\lambda}) - r_S]}, \quad (3.12)$$

respectively.

From 3.11 it is possible to see that CDSs work as insurance to both risky and safe lenders. Without CDSs the probability that lenders buy CDS is zero, which results in a loss of  $L(a)$  to both types of lenders if the borrower is in distress. With credit default swaps, lenders receive CDS payments when coordination succeeds. This reduces their expected loss since  $L_g(\frac{a}{\lambda}) < L(a)$ . It is also clear that risky lenders benefit more from CDSs insurance since  $L_R(\frac{a}{\lambda}) < L_S(\frac{a}{\lambda})$ . Equation 3.11 also implies that the payoffs of both types of lenders are increasing in the probability of a credit event, with that of risky lenders increasing faster.

Finally, it is also readily available from 3.12 that the liquidation cutoff is decreasing in the

repayments of each group. The reason is that a higher repayment reduces the opportunity cost of insuring, which facilitates coordination among lenders.

### 3.4 Optimal Financing Policy

In this section I examine the borrower's optional financing policy. In the presence of CDSs, the borrower needs to take into account that the probability of liquidation once in distress depends on proportion of debt hold by each type of lender. The next subsection derives the optimal financing policy in the absence of CDSs. This will serve as a benchmark once I derive the results in an economy with CDS markets.

In what follows, I assume  $\frac{C_R}{B_R} \leq \frac{C_S}{B_S}$  for all combinations of  $r_R$ ,  $r_S$ ,  $a$ , and  $\lambda$ . From 3.12, this implies that riskier lenders reduce the liquidation cutoff whereas safer lenders contribute increase this threshold. It follows that the higher the proportion of riskier lenders holding the borrower's debt, the higher the probability of a credit event given distress.

#### 3.4.1 Without CDSs

Without CDSs, the borrower need not worry about being liquidated if he is in distress. This implies that his only goal is to finance his debt with the smallest face value. The borrower's

problem in  $t = 0$  is the following:

$$\max_{r_g, p_g \in [0,1] \text{ for } g=R,S} (1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) + \lambda \int_0^\infty \theta h(\theta) d\theta$$

*s.t.*

$$p_R \beta + p_S (1 - \beta) = \alpha \quad (3.13)$$

$$p_g U_g(r_g) \geq 0 \quad \text{for all } g = R, S \quad (3.14)$$

The first constraint means that the borrower needs to borrow enough to finance his debt. The second constraints are lenders' participation constraints. It insures that lenders receive at least the same return they obtain if they choose not to finance the borrower and to store the money at the market interest rate (which is normalized to zero).

Since there is no insurance, lenders receive no additional payments when the borrower is in distress (except for the return on their illiquid assets  $a$ ). Because the loss functions are normalized such that both types of lenders lose the same amount  $L(a)$ , the heterogeneity among lenders is irrelevant. This intuition is formalized in Proposition 3 below.

**Proposition 3** *The optimal financing policy without CDSs has repayment  $r^* = \frac{1+\lambda L(a)}{1-\lambda}$  and probabilities of financing  $(p_R^*, p_S^*) = \left(p_R, \frac{\alpha - \beta p_R}{1-\beta}\right)$  for  $p_R \in \left[0, \frac{\alpha}{\beta}\right]$ . This contract generates a surplus to the borrower equal to*

$$W^*(\lambda) = (1 - \lambda) \left( \bar{y} - \alpha \frac{1 + \lambda L(a)}{1 - \lambda} \right) + \lambda \int_0^\infty \theta h(\theta) d\theta \quad (3.15)$$

Proposition 3 states that the borrower is indifferent between both types of lenders when he seeks finance as both types of lenders yield the same repayment  $r^* = \frac{1+\lambda L(a)}{1-\lambda}$ . This repayment is increasing in  $\lambda$  for two reasons. The first is because lenders' payoffs are decreasing

in the probability of distress. The second is that lenders' payoff are satisfied with equality in equilibrium. Finally, it is worth noting that the borrower's payoff is decreasing in  $\lambda$  as one of the assumptions is that  $\bar{y}$  is sufficiently large.

### 3.4.2 With CDSs

In the presence of CDS markets, financing terms and the proportion of each type of lender that holds the borrower's debt significantly impact the probability of distress given default. The borrower's problem in  $t = 0$  is the following:

$$\max_{r_g, p_g \in [0,1] \text{ for } g=R,S} (1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) + \lambda \int_{[p_R \beta r_R + p_S (1 - \beta) r_S](1 - \phi^*)}^{\infty} \theta h(\theta) d\theta$$

$s.t.$

$$p_R \beta + p_S (1 - \beta) = \alpha \tag{3.16}$$

$$p_g U_g(r_g) \geq 0 \quad \text{for all } g = R, S \tag{3.17}$$

The borrower's payoff with CDSs is clearly smaller under distress. However, this negative impact can be compensated by better financing terms and higher payoffs in non-distress times. Proposition 4 below shows that when the borrower's probability of distress is sufficiently small, the surplus brought about by CDSs is positive.

**Proposition 4** *Debt repayments with CDSs and known lenders are lower than those without CDSs. Let  $W^{**}(\lambda)$  be the borrower's surplus with CDSs and known lenders. There exists a cutoff  $\underline{\lambda}$  for the borrower's probability of distress such that, if  $\lambda > \underline{\lambda}$ , then  $W^{**}(\lambda) \leq W^*(\lambda)$  and if  $\lambda < \underline{\lambda}$ ,  $W^{**}(\lambda) \geq W^*(\lambda)$ . That is, the surplus brought about by CDSs is positive for safer borrowers and negative for riskier ones.*

The intuition behind this proposition is the following. On the one hand, CDSs work as insurance to lenders in the event the borrower is in distress, increasing their payoffs. This

allows the borrower to obtain finance with better terms, i.e., with lower repayments. On the other hand, CDSs increase lenders' outside options in distress times, causing the borrower to be liquidated with positive probability. If the distress probability is sufficiently low, the former effect dominates the latter. In this case, the surplus brought about by CDSs is positive. However, as the probability of distress increases, the CDS fee also increases, making insurance more costly. If the probability of distress is sufficiently high, the latter effect dominates the former, implying the borrower's surplus with CDSs is negative.

The next step is to understand how the borrower adjusts his financing policy according to his probability of distress. Proposition 5 states that safer borrowers lean more heavily on riskier lenders.

**Proposition 5** *The higher the borrower's probability of distress  $\lambda$ , the higher the repayment of lenders of group  $R$ , the lower the repayment of lenders of group  $S$ , and the lower the probability that lenders of group  $R$  finance the borrower.*

Proposition 5 states that the borrower weighs the benefits of reduced repayments in good times against the costs of a higher probability of liquidation in distress times. Since risky lenders benefit the most from CDS insurance, they allow for better financing terms compared to safe lenders. Thus, the borrower can extract a higher surplus by obtaining a larger fraction of finance from these lenders. The borrower can obtain even better financing terms with risky lenders by increasing the repayment of safe lenders. This is possible because a higher repayment for safe lenders increase these lenders' benefit from insurance. As a result, coordination among all lenders is facilitated, resulting in lenders buying CDS more often and in a higher probability of liquidation. The benefit of risky lenders from CDS insurance is further increased, lowering the borrower's financing cost.

The problem is that risky lenders contribute to a lower liquidation cutoff given distress. The more the borrower obtains funding from these lenders, the higher the probability of liquidation given distress. The benefits of lower repayments relative to increased distress costs is higher for safer lenders. Therefore, safer borrowers obtain a large portion of funding from risky lenders, face lower financing costs — although the spread between the highest and lowest repayments is higher, and have CDSs written more often on their debt.

Note that Proposition 5 provides a simple rationale to recent evidence provided by the empirical literature on CDSs. Ashcraft and Santos (2009) and Hirtle (2009) show that CDSs are more liquid for safer, larger firms and that these firms have benefited the most in terms of reduced interest on their bank loans. This result would seem puzzling at first glance since one would expect risky firms to benefit the most from CDSs in terms of greater credit supply and better financing terms. According to the model, however, safer firms benefit more from CDSs and lean more heavily on lenders that profit the most from CDS insurance, obtaining better financing terms. This results in lenders demanding protection more often, causing CDSs associated with safer firms to be more liquid.

## 3.5 Extensions

In this section, I extend the model to examine the impact of fees on the equilibrium and the implications of lenders' types being unknown to the borrower. In the first case, I show that a simple fee scheme in which lenders with higher distress exposure pay higher fixed fees to buy CDS protection would make CDSs beneficial to riskier borrowers. I then compare this fee scheme with the following commonly proposed interventions: limiting voting in restructuring

decisions to lenders with positive net economic ownerships — CDS protection below the amount of debt owed by the firm, making voluntary renegotiations trigger a credit event, and capping the amount of protection to the amount of debt owed by the borrower. The first two proposals are equivalent in terms of their impact according to the present model.

The first proposal has received wide attention in the press (see Mengle (2009)) and is supported by the legal scholars Hu and Black (2008a,b), who have proposed the empty creditor hypothesis. The second proposal has recently received some attention as CDS buyers have expressed concerns over the ability that entities have to avoid credit events.<sup>12</sup> The third proposal is suggested by Spamann (2010) and Bolton and Oehmke (2011) in order to avoid the empty voting and empty creditor problems, respectively.

I conclude that the fee system outperforms these proposals if the borrower’s probability of distress is sufficiently high — exactly the region for which CDSs can be harmful. In the second case I show that, if the borrower is unknown about lenders’ exposure to distress risk, then the benefit of CDSs is reduced.

### 3.5.1 CDS fees

Much has been proposed in terms of improving the efficiency of CDS markets. In this section, I show how a fixed fee scheme denoted by  $\tau_g$  for  $g = R, S$  affects the liquidation threshold given distress and find a fee incidence that improves the outcome for borrowers that are harmed by CDSs. Proposition 6 below shows that a fixed fee incidence enters linearly in the liquidation cutoff equation.

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<sup>12</sup>See “Restructuring credit event may be reviewed - ISDA,” Reuters, August 3, 2011.



**Proposition 6** *Suppose lenders of group  $R$  have to pay a fixed tax  $\tau_R$  if they buy CDSs. The equilibrium of the game that starts in  $t = 1$  is characterized by the following liquidation cutoff:*

$$\phi^*(\tau_R, \tau_S) = p_R \beta \frac{a + \lambda [L_R(r_R) - L(a) - r_R] + \tau_R}{a + \lambda [L_R(r_R) - L_R(\frac{a}{\lambda}) - r_R]} + p_S (1 - \beta) \frac{a + \lambda [L_S(r_S) - L(a) - r_S] + \tau_S}{a + \lambda [L_S(r_S) - L_S(\frac{a}{\lambda}) - r_S]}$$

Proposition 6 states that applying a fixed fee on lenders that buy CDS reduce the incremental benefit of insuring over not insuring, making coordination among lenders harder and increasing the liquidation cutoff. Since the liquidation costs caused by the introduction of CDSs are greater than its benefits for riskier borrowers, Proposition 6 suggests that imposing fees on CDS trading could improve the welfare of these borrowers.

A desirable fee scheme would reduce inefficient liquidation without eliminating the insurance property of CDSs. One intuitive way of accomplishing that is to allow liquidations only when the borrower's realized cash flow under distress is 0, i.e.,  $\theta \leq 0$ . In this way, the borrower loses nothing in distress times and lenders still benefit from CDSs. The resulting outcome is definitely higher than in the absence of CDS markets. It follows that imposing trading fees makes CDS beneficial to riskier lenders that would be harmed otherwise.

Proposition 7 formalizes this idea.

**Proposition 7** *Suppose the borrower's surplus with CDSs and known lenders is smaller than that without CDSs. Suppose further that the planner imposes fees on CDSs such that  $\tau_g = \lambda [L(a) - L_g(\frac{a}{\lambda})]$  and does not redistribute the fee revenue. Then the borrower's surplus with CDSs, known lenders, and fees is higher than that without CDSs.*

By choosing a fee scheme as in Proposition 7, the planner can induce a liquidation cutoff of  $\phi^* = 1$ . In other words, the borrower will be liquidated once in distress if and only if

$\phi = 1 - \frac{\theta}{\bar{r}} > 1$ , which is true if and only if the fundamental  $\theta$  is negative. A good property of this fee system is that it does not require information regarding lenders' private information and their distributions or the proportion of each type of lender holding the borrower's debt. It only requires a measure of the exposure of each group of lenders to borrower's distress and the probability of distress, where the former can be obtained from balance sheet data and the latter from CDS spreads. This fee scheme is also intuitive as the fee is higher if the CDS is written on a risky borrower and if the protection buyer is a risky lender.

### **3.5.1.1 Restructuring as a credit event**

Some of the applicable credit events are bankruptcy, obligation default, obligation acceleration, failure to pay, and repudiation/moratorium. The standard CDS contract, as defined by the International Swaps and Derivatives Association (ISDA), does not recognize out-of-court restructuring as a credit event. Although it is possible to include restructuring as a credit event in a CDS contract, CDS payment is triggered only if the restructuring binds all debt holders. As a result, a private workout in the U.S. corporate segment has never triggered a credit event. This has put pressure on the ISDA to broaden the restructuring definition such that to allow private workouts to constitute credit events. I examine the consequences of including voluntary renegotiations as a credit event and compare it to the fee scheme derived in the former subsection.

If voluntary restructurings are considered credit events, then the borrower is not liquidated under distress if and only if he is able to repay his debt in full, i.e.,  $y(\theta) < \bar{r}$ . It follows that each lender's decision to buy CDS protection does not depend on the overall fraction

of lenders that insure. Since lenders' signals are nearly precise, they will buy CDS if  $\theta < \bar{r}$  and will not buy if otherwise. Analogously, the borrower's financing policy need only worry about obtaining the best financing terms — as this also insures the lowest liquidation threshold. To be more precise, the borrower's payoff under restructuring as a credit event is

$$(1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) + \lambda \int_{[p_R \beta r_R + p_S (1 - \beta) r_S]}^{\infty} \theta h(\theta) d\theta, \quad (3.18)$$

while lenders' payoff are given by:

$$(1 - \lambda) r_g - \lambda \left[ H(\bar{r}) L_g \left( \frac{a}{\lambda} \right) + (1 - H(\bar{r})) L(a) \right] - 1. \quad (3.19)$$

Under the tax system proposed in the previous subsection, the liquidation cutoff is  $\phi^* = 1$ . Therefore, the borrower's payoff is

$$(1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) + \lambda \int_0^{\infty} \theta h(\theta) d\theta, \quad (3.20)$$

and the payoff of lenders of group  $g$  is

$$(1 - \lambda) r_g - \lambda \left[ H(0) L_g \left( \frac{a - \tau_g}{\lambda} + \tau_g \right) + (1 - H(0)) L(a) \right] - 1. \quad (3.21)$$

From 3.19 and 3.21 it is possible to conclude that lenders' payoff are higher under restructuring as a credit event. The reason is that this contract structure eliminates strategic uncertainty and solves lenders' coordination problem. Lenders become better protected in distress times, allowing the borrower to obtain better financing terms. However, this comes at a cost. The borrower will be liquidated more often compared to the fee scheme scenario. It follows that if the probability of borrower's distress is sufficiently small, restructuring as a

credit event is preferred to the fee scheme. However, the likelihood of distress is high enough, then the fee system is preferred. This is formalized in Proposition 8 below.

**Proposition 8** *Debt repayments are lower if the intervention takes the form of requiring restructuring to be a credit event as opposed to implementing a fixed fee scheme. Let  $W^r(\lambda)$  be the borrower's surplus in the former and  $W^\tau(\lambda)$  that in the latter. There exists a cutoff  $\bar{\lambda}$  for the borrower's probability of distress such that, if  $\lambda > \bar{\lambda}$ , then  $W^r(\lambda) \leq W^\tau(\lambda)$  and if  $\lambda < \bar{\lambda}$ ,  $W^r(\lambda) \geq W^\tau(\lambda)$ . That is, implementing the fee scheme is more (less) beneficial to riskier (safer) borrowers compared requiring restructuring to be a credit event.*

Proposition 8 states that, if the purpose of intervening in the CDS market is to make CDSs beneficial so riskier borrowers, then the fixed fee scheme proposed is preferred. The intuition is quite simple. The main problem faced by riskier borrowers with the existence of CDSs is that of frequent liquidation. Therefore, in order to overcome this problem, it is necessary to make it harder to lenders to coordinate when they buy CDS protection. Requiring restructuring as a credit event does exactly the opposite.

### 3.5.1.2 Capping CDS insurance

Capping the amount of CDS insurance to the amount of debt held by the lender has been proposed on theoretical grounds by Bolton and Oehmke (2011) and Spamann (2010). Bolton and Oehmke (2011) argue that the empty creditor problem is associated with negative economic ownership — CDS protection greater than renegotiation proceeds consistent with no liquidation. They suggest limiting the enforcement of excessively large default payments. Spamann (2010) shows that an informed trader such as a hedge fund destroys social value when he is overinsured — votes to minimize the security value (empty voting). It follows that capping the amount of CDS protection is a natural way of reducing inefficiencies in his model.

In the two papers mentioned in the last paragraph, the empty creditor and voting outcomes result from the action of a player that is big relative to the market. In other words, the inefficiency results from the following: (i) the player has negative economic ownership, i.e., prefers the inefficient outcome and (ii) the player is big enough so as to influence the outcome in his favor. Therefore, restricting the economic positions of these players to be non-negative improves welfare.

In the present paper, lenders are small relative to the market and face a coordination problem. Liquidation occurs only if the average belief about the total fraction of insured lenders is sufficiently high. The effect of capping the amount of CDS insurance to the amount of debt held by the lender facilitates lenders' coordination. The reason is that this intervention equalizes lenders' incremental payoff of insuring over not insuring regardless of whether a credit event occurs. Since it is dominant for lenders of group  $g$  to buy an amount of protection of at least  $r_g$ , this constraint on the amount of CDS protection results in lenders of group  $g$  buying exactly  $s_g = r_g$ . Lenders insure if  $\theta < \bar{r}$  and do not insure if otherwise, which means a higher probability of default given distress compared to the baseline model.

In terms of liquidation outcomes given distress, this policy is similar to that of requiring restructuring to constitute a credit event. As a result, the borrower's payoff following financing is exactly equal to that in 3.18. However, the surplus of lenders of group  $g$  is given by:

$$(1 - \lambda) r_g - \lambda [H(\bar{r}) L_g(r_g + a - \lambda r_g) + (1 - H(\bar{r})) L(a)] - 1. \quad (3.22)$$

If  $a$  is sufficiently high, these payoffs are smaller than the correspondent ones under the requirement that restructuring be a credit event. As a result, since for riskier borrowers the

fixed fee scheme is superior to the requirement that restructuring be a credit event, which in turn is superior to capping CDS insurance, I conclude that the proposed fixed tax system is the most appropriate if the goal is to make CDS beneficial to these borrowers.

### 3.5.2 Unknown lenders

So far I have assumed that the borrower knows the group of each lender. As a consequence, the borrower trades off increasing the proportion of risky lenders and obtaining better financing terms against increasing the proportion of safe lenders and inducing a lower probability of liquidation given distress. The safer the borrower, the more advantageous the former option. In addition, according to Proposition 5, this involves reducing the repayment of risky lenders and increasing the repayment of safe ones. In other words, the spread — the repayment of safe lenders minus that of risky ones — is higher for safer borrowers.

However, the assumption that lenders' exposures are known to the borrower might be somewhat unrealistic. There is large evidence that banks manipulate reports on their loan loss data (see Wall and Koch (2000) and Hasan and Wall (2004)). There is also evidence that banks delay loan provisioning (Laeven and Majnoni (2003)) and overstate the value of distressed assets (Huizinga and Laeven (2009)).

If lenders' types are unknown to the borrower, then risky lenders would have incentives to act strategically and conceal his type from the borrower. Therefore, the borrower just offer contracts that satisfy the lenders' incentive constraint, i.e., the borrower's policy must be such that lenders of one type do not prefer to be treated as those of the other type.

The new problem faced by the borrower in  $t = 0$  is the following:

$$\max_{r_g, p_g \in [0,1] \text{ for } g=R,S} (1 - \lambda) (\bar{y} - p_R \beta r_R - p_S (1 - \beta) r_S) + \lambda \int_{[p_R \beta r_R + p_S (1 - \beta) r_S](1 - \phi^*)}^{\infty} \theta h(\theta) d\theta$$

*s.t.*

$$p_R \beta + p_S (1 - \beta) = \alpha \quad (3.23)$$

$$p_g U_g(r_g) \geq 0 \quad \text{for all } g = R, S \quad (3.24)$$

$$p_g U_g(r_g) \geq p_{-g} U_g(r_{-g}) \quad \text{for all } g = R, S \quad (3.25)$$

As it is usual in this kind of problems, one can show that the only participation constraint that binds is that of the safe borrowers, while the only incentive constraint binding in the optimum is that of risky lenders. As Proposition 9 below states, the presence of unknown lenders makes the borrower increase the fraction of funds obtained from safe lenders, decrease the repayment of safe lenders, and increase the repayment of risky lenders.

**Proposition 9** *With CDSs and unknown lenders, the repayment of lenders of group R is higher, the repayment of lenders of group S is lower, and the probability that lenders of group R finance the borrower is lower compared to the case with CDSs and known lenders.*

Proposition 9 says that the existence of unknown lenders makes it less attractive to obtain funding from risky lenders. When lenders' types are known, the borrower obtains the best lending terms from risky lenders. This benefit is weighed against the cost of facing a higher probability of liquidation given distress. With unknown lenders, the borrower must make financing terms more favorable to risky lenders since otherwise they prefer the contracts offered to safe lenders. Therefore, it makes financing risky lenders relatively more costly compared to safe lenders. This suggests that a good way to improve the market

would be to obtain good measures of lenders exposure the borrowers' distress and to make this information accessible to potential borrowers.

### 3.6 Empirical Implications

I dedicate this section to the discussion of the model's implications. I present a non-exhaustive list of testable empirical predictions. I believe that examining these predictions would deepen our understanding of the CDS markets and their impact on corporate financing and economic efficiency.

*Implication #1: Firms with a greater proportion of debt held by lenders that are more exposed to distress should experience a higher probability of default given distress.*

This result follows from Proposition 2. Riskier lenders reduce the liquidation cutoff whereas safer lenders contribute increase this threshold. Therefore, the higher the proportion of riskier lenders holding the firms's debt, the higher the probability of default given distress.

*Implication #2: Among firms with similar distress probability and debt composition, those with higher debt repayments should experience a higher probability of default given distress.*

This result also follows from Proposition 2. The lower the debt repayments, the higher the opportunity of insuring. This makes coordination among lenders more difficult, increasing the liquidation cutoff.

*Implication #3: Firms should experience better financing terms after the emergence of CDSs*

This result follows from Proposition 4. CDSs work as insurance to lenders in the event



the firm is in distress, increasing their payoffs. This allows firms to obtain finance with better terms, i.e., with lower repayments.

*Implication #4: The equity value of safer firms should increase after the emergence of CDSs while that of riskier ones should decrease.*

This implication also follows from Proposition 4. CDSs allows firms to obtain lower repayments at the cost of increasing the probability liquidation given distress. The former effect dominates the latter for safer firms while the converse is true for riskier firms.

*Implication #5: Safer firms should experience better financing terms and higher probability of default given distress after the emergence of CDSs.*

This implication follows from Proposition 5. The benefits of lower repayments relative to increased distress costs is higher for safer lenders. Therefore, safer firms face lower financing costs. This comes at the cost of having CDSs written more often on their debt, which increases the probability of default given distress.

*Implication #6: Safer firms should have a greater proportion of debt held by lenders that are more exposed to distress.*

This result also follows from Proposition 5. Lenders that are highly exposed benefit the most from CDS insurance, allowing the firm to finance his debt offering lower repayments. Increasing the proportion of these lenders reduce the overall face value of debt but decreases the liquidation cutoff. Therefore, safer borrowers finance more heavily with more exposed lenders

While my model's predictions are new and have not been directly taken to the data, some reported empirical regularities are consistent with my theory. I argue, for example,

that CDSs are more beneficial for firms that are safer. This result is interesting and stands in contrast to common intuition that riskier firms would benefit the most from the existence of CDS markets. Consistent with my theory, however, recent studies by Ashcraft and Santos (2009) and Hirtle (2009) find that safer and larger firms have benefited the most from CDS contracts (for example, by paying lower spreads on their bank loans). Song and Uzmanoglu (2010) explore the financial crisis and find evidence that firms associated with safer banks observed smaller CDS spreads in the 2008–9 period.

A number of other predictions listed above can be directly taken to the data. Empirical research on CDS is still in its infancy and this strikes us as setting in which models describing rich sets of relations — some direct, others indirect — are particularly useful in guiding empirical work.

### 3.7 Concluding Remarks

The emergence of CDSs could be considered an important source of improvements to capital markets. As Alan Greenspan once noted, “The new instruments of risk dispersion have enabled the largest and most sophisticated banks in their credit-granting role to divest themselves of much credit risk by passing it to institutions with far less leverage.”<sup>13</sup> By promoting better risk sharing, CDSs could allow for greater credit supply and better terms for firms. Theoretically, riskier firms would benefit the most from the existence of CDS insurance. This is in contrast with recent evidence by Ashcraft and Santos (2009) and Hirtle (2009) that find that CDSs are more liquid for safer, larger firms and that these firms have

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<sup>13</sup>From his speech “Economic Flexibility” before Her Majesty’s Treasury Enterprise Conference (London, 26 January, 2004).

benefited the most from CDS markets. This puzzling result suggests that CDSs can be beneficial to some firms and less so to others.

This paper provides a simple rationale to this apparent puzzle by developing a model examining the optimal financing policy in the presence of CDSs when competitive lenders have different exposures to borrowers' distress risk and face coordination problems. The model shows that the borrower weighs the benefits of reduced repayments in good times against the costs of a higher probability of liquidation in distress times. Since risky lenders benefit the most from CDS insurance, they allow for better financing terms compared to safe lenders. Thus, the borrower can extract a higher surplus by obtaining a larger fraction of finance from these lenders.

The problem is that risky lenders contribute to a lower liquidation cutoff given distress. The more the borrower obtains funding from these lenders, the higher the probability of liquidation given distress. The benefits of lower repayments relative to increased distress costs is higher for safer lenders. Therefore, safer borrowers obtain a large portion of funding from risky lenders, face lower financing costs — although the spread between the highest and lowest repayments is higher, and have CDSs written more often on their debt. Overall, firms that are sufficiently safe benefit from CDSs as the surplus brought about in terms of reduced repayments in good times surpasses the extra distress costs. On the other end of the spectrum, firms for which distress probabilities are large enough have inefficiently low liquidation cutoffs. These firms are frequently liquidated once in distress and CDSs are harmful to them.

I look for policy innovations that might improve efficiency. I show that a simple fee scheme in which lenders pay a fixed fee in order to buy CDS protection can reduce inefficient

liquidation. The fees are proportional to lenders' exposure, with the proportionality constant given by the borrower's probability of distress. In other words, fees are higher for CDSs written on the debt of riskier borrowers and for lenders that are more exposed to the borrower's risk of distress. As a result, it becomes harder for lenders to coordinate, which raises the liquidation cutoff. This fee system requires very little knowledge from the planner. It does not require information regarding lenders' private information and their distributions or the proportion of each type of lender holding the borrower's debt. It only requires a measure of the exposure of each group of lenders to borrower's distress and the probability of distress, where the former can be obtained from balance sheet data and the latter from CDS spreads.

I compare this fee scheme with commonly proposed interventions such as limiting voting in restructuring decisions to lenders with positive net economic ownerships and capping the amount of protection to the amount of debt owed by the borrower. I show that the fee system is superior to these proposals if the borrower's probability of distress is sufficiently high, which turns out to be the range for which CDSs can be harmful otherwise. The reason is that these proposals work as a coordination tool to lenders, resulting in an excessively low liquidation cutoff and a high distress cost.

If lenders' exposures are unknown to the borrower at the time of financing (e.g., lenders' distress risk is hard to measure precisely), more exposed lenders want to look like less exposed ones in order to enjoy higher repayments. This harms specially safer borrowers since they lean more heavily on more exposed lenders. As a result, these borrowers inefficiently increase the proportion of less exposed lenders, reducing the surplus brought about by CDSs. Proposals aimed at making information regarding lenders' risk exposure more transparent,

accessible, and accurate could greatly improve market efficiency.

## 3.8 Figures and Tables

**Table 3.1:** Timing

$t = 0$ (Investment Period)	$t = 1$ (CDS Insurance)	$t = 2$ (Renegotiation)
1. Borrower offers contracts $(p_g, r_g)$ .	1. Lenders receive signals $x_i$ about the fundamental $\theta$ 2. Lenders buy CDS protection.	1. Cash flow $c$ is realized. If $c = \bar{y}$ no default. 2. If $c = y(\theta)$ , renegotiation.

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# Appendix A

## Proofs of Chapter 1

**Proof of Lemma 1** The expected payoff of the VC under uninformed financing at the time of refinancing is  $v_{\iota_V}^U p (\pi_H - R_1^U) + (1 - v_{\iota_V}^U) \bar{u}$ . Therefore,  $v_{\iota_V}^U = 1$  if and only if  $p (\pi_H - R_1^U) \geq \bar{u} \iff \pi_H - \frac{\bar{u}}{p} \geq R_1^U$ . Analogously, the expected payoff of the VC at the time of refinancing under informed financing is  $v_{\iota_V}^I p (\pi_H - R_1^I) + (1 - e_{\iota_V}^I) \bar{u}$ . Therefore  $e_{\iota_V}^I = 1$  if and only if  $\pi_H - \frac{\bar{u}}{p} \geq R_1^I$ .

The payoff of the investor under uninformed financing is  $v_{\iota_V}^2 (pR_1^U - 1)$ , which is equal to 0 if  $R_1^U > \pi_H - \frac{\bar{u}}{p}$  and equal to  $pR_1^U - 1$  for  $R_1^U \leq \pi_H - \frac{\bar{u}}{p}$ . The optimal strategy is  $R_1^U = \pi_H - \frac{\bar{u}}{p}$ , which gives a payoff of  $p\pi_H - \bar{u} - 1 > 0$ . The investor's payoff under informed financing is  $v_{\iota_V}^I (pR_1^I - 1)$ , which is equal to 0 if  $R_1^I > \pi_H - \frac{\bar{u}}{p}$  and  $pR_1^I - 1$  if otherwise. It is optimal for the investor to choose  $R_1^I = \pi_H - \frac{\bar{u}}{p}$ , which gives him a payoff of  $p\pi_H - \bar{u} - 1 > 0$ .

**Proof of Lemma 2** Suppose there is a VC-equilibrium each VC charges a different price  $z_{\iota_V}$ . We will show that the best strategy for the investor is to choose  $k = 0$ , which leads to a contradiction. Note that, by consistency of beliefs, we have  $\mu_K(\iota_V | z_{\iota_V}) = \mu_K(\iota_V | z_{\iota_V}, \tau_{\iota_V}) = 1$  and  $\mu_E(\iota_V | \iota_E, z_{\iota_V}) = \mu_E(\iota_V | \iota_E, z_{\iota_V}, \tau_{\iota_V}) = 1$ . Using this along with the result of lemma 1

will allow us to compare the investor's payoff when  $k = 0$  to that when  $k = 1$ . The entrepreneur's belief that his project is of type  $G$  is  $\mu(G|\iota_E, \iota_V)$ . Without loss of generality, assume  $\mu(G|\iota_E, \iota_V) > 0$ .

**Case 1 ( $k = 0$ ):** We can write the entrepreneur's expected payoff by:

$$e_{\iota_E}^U \mu(G|\iota_E, \iota_V) p(\pi_H - R_0^U) + (1 - e_{\iota_E}^U)(m_H + \bar{u}).$$

Therefore, she will choose  $e_{\iota_E}^U = 1$  if and only if:

$$\mu(G|\iota_E, \iota_V) p(\pi_H - R_0^U) \geq m_H + \bar{u}$$

Since the investor knows that type of the VC, his belief that the project is good is  $\mu(G|\iota_V)$ .

The expected payoff of the investor is:

$$e_{\iota_E}^U (\mu(G|\iota_V) p R_0^U - (1 - m_H)) + e_{\iota_E}^U p \mu(G|\iota_V) (p\pi_H - \bar{u} - 1).$$

Thus, if he sets  $R_0^U \leq \pi_H - \frac{m_H + \bar{u}}{\mu(G|\iota_E, \iota_V)p}$  his expected payoff is

$$\mu(G|\iota_V) p R_0^U - (1 - m_H) + p \mu(G|\iota_V) (p\pi_H - \bar{u} - 1), \quad (\text{A.1})$$

and if he sets otherwise, 0. He will choose  $R_0^U = \pi_H - \frac{m_H + \bar{u}}{\mu(G|\iota_E, \iota_V)p}$  if and only if:

$$\mu(G|\iota_V) p \pi_H - (m_H + \bar{u}) \frac{\mu(G|\iota_V)}{\mu(G|\iota_E, \iota_V)} - (1 - m_H) + p \mu(G|\iota_V) (p\pi_H - \bar{u} - 1) \geq 0. \quad (\text{A.2})$$

**Case 2 ( $k = 1$ ):** The entrepreneur's expected payoff is:

$$e_{\iota_E}^I \mu(G|\iota_E, \iota_V) p(\pi_H - R_0^I) + (1 - e_{\iota_E}^I)(m_H + \bar{u}).$$

This implies that the entrepreneur will choose  $e_{\iota_E}^I = 1$  if and only if:

$$\mu(G|\iota_E, \iota_V) p (\pi_H - R_0^I) \geq m_H + \bar{u}$$

The investor's expected payoff is:

$$e_{\iota_E}^I (\mu(G|\iota_V) p R_0^I - (1 - m_H) - z_{\iota_V}) - (1 - e_{\iota_E}^I) z_{\iota_V} + e_{\iota_E}^I p \mu(G|\iota_V) (p\pi_H - \bar{u} - 1).$$

If he chooses  $R_0^I \leq \pi_H - \frac{m_H + \bar{u}}{\mu(G|\iota_E, \iota_V)p}$  his expected payoff is

$$\mu(G|\iota_V) p R_0^I - (1 - m_H) - z_{\iota_V} + p \mu(G|\iota_V) (p\pi_H - \bar{u} - 1), \quad (\text{A.3})$$

and if he sets otherwise,  $-z_{\iota_V}$ . He will choose  $R_1 = \pi_H - \frac{m_H + \bar{u}}{\mu(G|\iota_E, \iota_V)p}$  if and only if:

$$\mu(G|\iota_V) p \pi_H - (m_H + \bar{u}) \frac{\mu(G|\iota_V)}{\mu(G|\iota_E, \iota_V)} - (1 - m_H) + p \mu(G|\iota_V) (p\pi_H - \bar{u} - 1) \geq 0. \quad (\text{A.4})$$

Note that (A.2)  $\iff$  (A.4) and (A.1)  $>$  (A.3). Thus, if (A.4) holds, then the investor is better off choosing  $k = 0$ . If (A.4) does not hold, then the investor's payoff if  $k = 1$  is  $-z_{\iota_V}$ , while his payoff if  $k = 0$  is  $0 > -z_{\iota_V}$ . Therefore, the investor chooses  $k = 0$  and we have a contradiction.

**Proof of Lemma 3** If  $k = 0$ , the entrepreneur of type  $\iota_E$  believes his project is of type

$G$  with probability  $\mu_{\iota_E}(G|\iota_E, z) \equiv \sum_{\iota'_V \in S(T)} \mu_{\iota_E}(\iota'_V|\iota_E, z) \mu(G|\iota'_V, \iota_E)$ . Therefore, his expected payoff is:

$$e_{\iota_E}^U \mu_{\iota_E}(G|\iota_{E1}, z) p (\pi_H - R_0) + (1 - e_{\iota_E}^U) (m_H + \bar{u}).$$

He will choose  $e_{\iota_E}^U = 1$  if and only if:

$$\mu_{\iota_E}(G|\iota_E, z) p(\pi_H - R_0) \geq m_H + \bar{u}.$$

If  $\iota_E = \{B\}$ , we have  $\mu_{\iota_E}(G|\iota_E, z) = 0$  and  $e_{\{B\}}^U = 0 \forall R_0^U \in \mathbb{R}^+$ . If  $\iota_E = \{G\}$ , then  $\mu_{\iota_E}(G|\iota_E, z) = 1$  and  $e_{\{G\}}^U = 1$  if and only if  $R_0^U \leq \pi_H - \frac{m_H + \bar{u}}{p} \equiv \bar{R}_0$ . If  $\iota_E = \{U\}$ , then  $\mu_{\iota_E}(G|\iota_E, z) = \lambda$  and  $e_{\{U\}}^U = 1$  if and only if  $R_0^U \leq \pi_H - \frac{m_H + \bar{u}}{\lambda p} \equiv \underline{R}_0^U$ . The argument is the same if  $k = 1$ , which concludes the proof.

**Proof of Lemma 4** One must note that, given the strategies chosen by the entrepreneurs, the expected payoff of the investor is increasing in  $R_0^U$ . If it is optimal for the investor to choose  $R_0^U \in [0, \underline{R}_0^U]$ , only the entrepreneurs of types  $\{U\}$  and  $\{G\}$  will accept the contract. Hence, the investor chooses  $R_0^U = \underline{R}_0^U$ . If it is optimal to choose  $R_0^U \in (\underline{R}_0^U, \bar{R}_0^U]$  only the entrepreneur of type  $\{G\}$  will accept the contract and the investor chooses  $R_0^U = \bar{R}_0^U$ . For  $R_0^U > \bar{R}_0^U$  no entrepreneur will accept the contract and the investor's expected return is  $d$ . In this case a contract with price  $R_0^U = \bar{R}_0^U$  gives the investor a higher payoff. To see this, note that the investor's belief that the entrepreneur is of type  $\iota_E$  is  $\mu_K(\iota_E|z) = \sum_{\iota'_V \in S(z)} \mu_K(\iota'_V|z) \mu(\iota_E|\iota'_V)$  the expected payoff of the investor if he charges  $\bar{R}_0^U$  is

$$\mu_K(\{G\}|z) [pR_0^U - (1 - m_H)] + \mu_K(\{G\}|z) p(p\pi_H - \bar{u} - 1) =$$

$$\mu_K(\{G\}|z) [p\pi_H(1 + p) - (m_H + \bar{u}) - 1] > d,$$

which is positive since  $\mu_K(\{G\}|z) = \lambda q$  and we assume  $\pi_H - \frac{m_H + \bar{u}}{\lambda p} > 1$ .

**Proof of Lemma 5** Suppose in equilibrium  $k = 1$ . If all types of VC report the same infor-



mation  $\tau$ , then consistency implies that  $\mu_K(\{G\}|z) = \mu_K(\{G\}|z, \tau) = \lambda q$ ,  $\mu_K(\{B\}|z) = \mu_K(\{B\}|z, \tau) = (1 - \lambda)q$ , and  $\mu_K(\{U\}|z) = \mu_K(\{U\}|z, \tau) = 1 - q$ . The implied beliefs for the entrepreneur are  $\mu_E(G|\{G\}, z) = \mu_E(G|\{G\}, z, \tau) = 1$ ,  $\mu_E(G|\{B\}, z) = \mu_E(G|\{B\}, z, \tau) = 0$ , and  $\mu_E(G|\{U\}, z) = \mu_E(G|\{U\}, z, \tau) = \lambda$ . Therefore, the same argument used in the proof of lemma 2 applies and the investor is better off choosing  $k = 0$ , which leads to a contradiction.

**Proof of Lemma 6** Suppose it is not; i.e.,  $1 > d - (1 - m_H) + R_0^i$ . Since an optimal debt contract has a price  $R_0^i \in \{\underline{R}_0^i, \overline{R}_0^i\}$ , the assumption  $\pi_H - \frac{m_H + \bar{u}}{\lambda p} > 1$  implies  $R_0^i > 1$ . Therefore,  $d - (1 - m_H) + R_0^i > 1 + d - (1 - m_H) \geq 1 > d - (1 - m_H) + R_0^i \Rightarrow 1 > R_0^i$ , which is a contradiction.

**Proof of Proposition 1** By lemma 4, we need only consider four types of equilibria:  $\tau_{\{G\}} \neq \tau_{\{U\}} \neq \tau_{\{B\}}$ ;  $\tau_{\{U\}} = \tau_{\{B\}} = \tau$ ,  $\tau_{\{G\}} \neq \tau$ ;  $\tau_{\{G\}} = \tau_{\{U\}} = \tau$ ,  $\tau_{\{B\}} \neq \tau$ ;  $\tau_{\{G\}} = \tau_{\{B\}} = \tau$ ,  $\tau_{\{U\}} \neq \tau$ . In order to characterize VC-equilibria, we need to find the optimal strategy for the investor given his beliefs, check if the reports given by VCs are optimal given the investor's beliefs, and see whether  $e_K = 1$  is optimal for the investor. The expected payoff of the VC of type  $\iota_V$  is  $\sum_{\iota_E \in T} e_{\iota_E}^I \mu(\iota_E|\iota_V) \mu(G|\iota_E, \iota_V) p \bar{u}$ . The investor's expected payoff is

$$\sum_{\iota_E \in T} \mu_K(\iota_E|z, \tau) \left[ \frac{e_{\iota_E}^I(R_1(\tau_{\iota_V})) (\mu(G|\iota_E) p R_0^I(\tau_{\iota_V}) - (1 - m_H) - z) - (1 - e_{\iota_E}^I(R_1(\tau_{\iota_V}))) z + e_{\iota_E}^I \mu(G|\iota_E) p (p \pi_H - \bar{u} - 1)}{1} \right].$$

**Case 1** ( $\tau_{\{G\}} \neq \tau_{\{U\}} \neq \tau_{\{B\}}$ ): Suppose  $\Pi(q = 0) > 0$ . In these equilibria  $\mu_K(\iota_V|z, \tau_{\iota_V}) = 1$ ,  $\mu_K(\iota_{E_1}|z, \tau_{\iota_V}) = \mu(\iota_E|\iota_V)$ . If  $\iota_{E_2} = \{G\}$ , then the investor clearly chooses  $\underline{R}_0^I(\tau_{\{G\}}) = \overline{R}_0^I$  and receives a payoff of  $\bar{\Pi}(\lambda = 1, q = 1) - z$ . If  $\iota_V = \{U\}$ , then  $\mu_E(G|\iota_E, z, \tau) = \lambda$  and

$\underline{R}_0^I(\tau_{\{U\}}) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ . The investor receives a payoff of  $-z$  if  $R_0^I > \underline{R}_0^I(\tau_{\{U\}})$  and a payoff of  $\underline{\Pi}(q=0) - z$  if  $R_0^I = \underline{R}_0^I(\tau_{\{U\}})$ . Since  $\underline{\Pi}(q=0) > 0$ , the investor chooses  $R_0^I(\tau_{\{U\}}) = \underline{R}_0^I(\tau_{\{U\}})$ . If  $\iota_V = \{B\}$ , then the investor's payoff is  $-z \forall R_0^I \in \mathbb{R}^+$ . The expected payoff of the VC of type  $\iota_V = \{G\}$  is  $p\bar{u}$  independent of his report, so he has no incentive to deviate. The expected payoff of the VC of type  $\iota_V = \{U\}$  is  $\lambda p\bar{u}$  if he reports  $\tau_{\{U\}}$  and at most  $\lambda p\bar{u}$  if he reports either  $\tau_{\{G\}}$  or  $\tau_{\{B\}}$ . Therefore, he has no incentive to deviate. The expected payoff of the VC of type  $\iota_V = \{B\}$  is 0 regardless of his report and he has no reason to deviate. The ex ante expected payoff of the investor is

$$\Pi = \bar{\Pi} + (1 - q) \underline{\Pi}(q=0) = \underline{\Pi} + q(1 - \lambda)(m_H + \bar{u}) > \max\{\bar{\Pi}, \underline{\Pi}\},$$

and the investor chooses  $k = 1$  if  $z \leq \min\{d - (1 - m_H), \Pi - \max\{\bar{\Pi}, \underline{\Pi}\}\}$ , and  $k = 0$  otherwise. Consequently, VCs will optimally set  $z = \min\{d - (1 - m_H), \Pi - \max\{\bar{\Pi}, \underline{\Pi}\}\}$  and we have a characterization of VC-equilibria when  $\underline{\Pi}(q=0) > 0$ .

Suppose now there is a VC equilibrium with  $\underline{\Pi}(q=0) \leq 0$ . If  $\underline{\Pi}(q=0) = 0$  and  $R_1(\tau_{\{U\}}) = \underline{R}_1(\tau_{\{U\}})$ , then we know a VC-equilibrium cannot exist since  $z = 0$  in this case, which leads to a contradiction. If  $R_0^I(\tau_{\{U\}}) > \underline{R}_0^I(\tau_{\{U\}})$ , then the VC of type  $\iota_V = \{U\}$  has an incentive to deviate and report  $\tau_{\{B\}}$  provided that  $R_0^I(\tau_{\{B\}}) \leq \underline{R}_0^I(\tau_{\{U\}})$ . Therefore, we must have  $R_0^I(\tau_{\{B\}}) > \underline{R}_0^I(\tau_{\{U\}})$  in order to sustain such equilibria. However, the investor's ex ante payoff under informed financing is  $\bar{\Pi} \leq \max\{\bar{\Pi}, \underline{\Pi}\}$ , which implies  $z = 0$  and we have a contradiction. If  $\underline{\Pi}(q=0) < 0$  then it must be that  $R_0^I(\tau_{\{U\}}) > \underline{R}_0^I(\tau_{\{U\}})$  and we already know that  $z = 0$ , leading to a contradiction. Hence, a VC-equilibrium exists if and only if  $\underline{\Pi}(q=0) > 0$ .

**Case 2** ( $\tau_{\{U\}} = \tau_{\{B\}} = \tau$ ,  $\tau_{\{G\}} \neq \tau$ ): Suppose  $\underline{\Pi}(q=0) > 0$ . In these equilibria, we have  $\mu_K(\{G\} | z, \tau_{\{G\}}) = 1$ ,  $\mu_K(\{U\} | z, \tau) = \frac{1-q}{1-q+(1-\lambda)q}$ , and  $\mu_K(\{B\} | z, \tau) = \frac{(1-\lambda)q}{1-q+(1-\lambda)q}$ . If  $\iota_V = \{G\}$ , then the investor chooses  $R_0^I(\tau_{\{IG\}}) = \overline{R}_1$ , receiving a payoff of  $\overline{\Pi}(\lambda=1, q=1) - z$ . Note that  $\mu_E(G | \{U\}, z, \tau) = \lambda$ , which implies  $\underline{R}_0^I(\tau) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ . If the investor chooses  $R_0^I > \underline{R}_0^I(\tau)$  upon receiving a report from either type  $\{U\}$  or type  $\{B\}$ , his payoff is  $-z$ . If he chooses  $R_0^I = \underline{R}_0^I(\tau)$  instead:

$$\frac{1-q}{1-q+(1-\lambda)q} (\underline{\Pi}(q=0) - z) - \frac{(1-\lambda)q}{1-q+(1-\lambda)q} z.$$

Therefore, the investor will choose  $R_0^I(\tau) = \underline{R}_0^I(\tau)$ . Clearly, no VC has an incentive to deviate regardless of  $R_0^I(\tau')$  for  $\tau' \in T \setminus \{\tau, \tau_{\{G\}}\}$ , which implies the investor can have any belief upon receiving report  $\tau'$ . As a consequence, the investor's ex ante payoff will be  $\overline{\Pi} + (1-q)\underline{\Pi}(q=0)$  and the investor chooses  $k=1$  if the management fee is such that  $z \leq \min\{d - (1-m_H), \Pi - \max\{\overline{\Pi}, \underline{\Pi}\}\}$ , and  $k=0$  otherwise. VCs charge  $z = \min\{d - (1-m_H), \Pi - \max\{\overline{\Pi}, \underline{\Pi}\}\}$ . Thus, we characterized the VC-equilibria when it holds that  $\underline{\Pi}(q=0) > 0$ . Suppose now there is a VC equilibrium with  $\underline{\Pi}(q=0) \leq 0$ . The argument that leads to a contradiction is that same as that in Case 1. Therefore, a VC-equilibrium exists if and only if  $\underline{\Pi}(q=0) > 0$ .

**Case 3** ( $\tau_{\{G\}} = \tau_{\{U\}} = \tau$ ,  $\tau_{\{B\}} \neq \tau$ ): Suppose  $\underline{\Pi}(q=0) > 0$ . In this equilibria, we have  $\mu_K(\{B\} | z, \tau_{\{B\}}) = 1$ ,  $\mu_K(\{U\} | z, \tau) = \frac{1-q}{1-q+\lambda q}$ , and  $\mu_K(\{G\} | z, \tau) = \frac{\lambda q}{1-q+\lambda q}$ . Since the uninformed entrepreneur has belief  $\mu_E(G | \{U\}, z, \tau) = \lambda$  that the holds a good project, if the investor chooses  $\underline{R}_0^I(\tau) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$  upon receiving a report from either type  $\{U\}$  or type  $\{G\}$ , his expected payoff will be  $\frac{\underline{\Pi}}{1-q+\lambda q} - z$ . If the investor chooses  $\overline{R}_0^I$  instead, his

expected payoff will be  $\frac{\bar{\Pi}}{1-q+\lambda q} - z$ . Therefore, if  $\bar{\Pi} > \underline{\Pi}$  the investor chooses  $R_0^I(\tau) = \bar{R}_0^I$ , and if  $\bar{\Pi} < \underline{\Pi}$  the investor chooses  $R_0^I(\tau) = \underline{R}_0^I(\tau)$ . Regarding a report from the VC of type  $\{B\}$ ,  $e_{\{B\}}^1 = 0$  and the investor's expected payoff is  $-z \forall R_0^I \in \mathbb{R}^+$ . The VC of type  $\{G\}$  has clearly no incentive to deviate. If  $\bar{\Pi} > \underline{\Pi}$  the VC of type  $\{U\}$  has an incentive to deviate provided that either  $R_0^I(\tau')$  or  $R_0^I(\tau_{\{B\}})$  is less than or equal to  $\underline{R}_0^I(\tau)$  for  $\tau' \in T \setminus \{\tau, \tau_{\{B\}}\}$ . In this case, has an expected payoff of 0 if he reports  $\tau$  and  $\lambda p \bar{u}$  otherwise. Thus, we must have  $R_0^I(\tau'), R_0^I(\tau_{\{B\}}) > \underline{R}_0^I(\tau)$  to sustain such equilibria. However, the investor's ex ante payoff under informed financing is  $\Pi = \bar{\Pi} \leq \max\{\bar{\Pi}, \underline{\Pi}\}$  and he is better off choosing  $k = 0$ . Hence, no VC-equilibrium exists if  $\bar{\Pi} > \underline{\Pi}$ . If  $\bar{\Pi} < \underline{\Pi}$  then no VC has incentive to deviate regardless of  $R_0^I(\tau')$  and  $R_0^I(\tau_{\{B\}})$ , which implies the investor can have any belief upon receiving report  $\tau'$ . However, the investor's ex ante payoff will be  $\Pi = \underline{\Pi} \leq \max\{\bar{\Pi}, \underline{\Pi}\}$  and he is better off choosing  $k = 0$ . Therefore, there is no VC-equilibrium if  $\bar{\Pi} < \underline{\Pi}$ . Conversely, there is no VC-equilibrium if  $\underline{\Pi}(q=0) \leq 0$  since in this case the investor must choose  $R_0^I(\tau) = \bar{R}_0^I$  and we know the implied investor's expected payoff is  $\Pi = \bar{\Pi} \leq \max\{\bar{\Pi}, \underline{\Pi}\}$ .

**Case 4** ( $\tau_{\{G\}} = \tau_{\{B\}} = \tau$ ,  $\tau_{\{U\}} \neq \tau$ ): Suppose  $\underline{\Pi}(q=0) > 0$ . In these equilibria, we have  $\mu_K(\{U\} | z, \tau_{\{U\}}) = 1$ ,  $\mu_K(\{B\} | z, \tau) = \frac{(1-\lambda)q}{(1-\lambda)q+\lambda q}$ , and  $\mu_K(\{G\} | z, \tau) = \frac{\lambda q}{(1-\lambda)q+\lambda q}$ . Since the uninformed entrepreneur has belief  $\mu_E(G | \{U\}, z, \tau_{\{U\}}) = \lambda$  that he has a good project,  $\underline{R}_0^I(\tau_{\{U\}}) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ . If the investor chooses  $R_0^I > \underline{R}_0^I(\tau_{\{U\}})$  upon receiving a report from the VC of type  $\{U\}$ , his expected payoff is  $-z$ . If he chooses  $R_0^I = \underline{R}_0^I(\tau_{\{U\}})$ , his expected payoff is  $\underline{\Pi}(q=0) - z$ . Therefore, the investor chooses  $R_0^I(\tau_{\{U\}}) = \underline{R}_0^I(\tau_{\{U\}})$ . The investor will clearly choose  $\bar{R}_0^I$  upon receiving the report  $\tau$ , which implies  $R_0^I(\tau) = \bar{R}_0^I$  and an associated payoff of  $\frac{\bar{\Pi}}{1-q+\lambda q} - z$ . It is straightforward to check that no VC has an incentive

to deviate regardless of  $R_0^I(\tau')$  for  $\tau' \in T \setminus \{\tau, \tau_{\{U\}}\}$ . Therefore, there are no restrictions on the investor's belief upon receiving report  $\tau'$ . The investor's ex ante payoff under informed financing is  $\Pi = \underline{\Pi} + (1 - q) \underline{\Pi}(q = 0) > \max\{\bar{\Pi}, \underline{\Pi}\}$ . The investor chooses  $k = 1$  if  $z \leq \min\{d - (1 - m_H), \Pi - \max\{\bar{\Pi}, \underline{\Pi}\}\}$  and  $k = 0$  if otherwise, and the VCs will optimally choose  $z = \min\{d - (1 - m_H), \Pi - \max\{\bar{\Pi}, \underline{\Pi}\}\}$ . We have characterized the VC-equilibria when  $\underline{\Pi}(q = 0) > 0$ . The argument to show that no VC-equilibrium exists if  $\underline{\Pi}(q = 0) \leq 0$  is the same as that in Case 1.

Therefore, we have shown that a VC-equilibrium exists if and only if  $\underline{\Pi}(q = 0) > 0$ . Moreover, we have characterized the VC-equilibria and they are all payoff-equivalent.

**Proof of Proposition 2** We start by showing (i). The payoff of the VC of type  $\iota_V$  is

$\sum_{\iota_E \in T} e_{\iota_E}^1 \mu(\iota_E | \iota_V) \mu(G | \iota_E, \iota_V) p \bar{u}$ , and the investor's expected payoff upon receiving a report from VCs is

$$\sum_{\iota_E \in T} \mu_K(\iota_E | z, \tau) \left[ \begin{array}{l} e_{\iota_E}^I(R_0^I(\tau_{\iota_V})) (\mu(G | \iota_E) p R_0^I(\tau_{\iota_V}) - (1 - m_H) - z) - \\ (1 - e_{\iota_E}^I(R_0^I(\tau_{\iota_V}))) z + e_{\iota_E}^I \mu(G | \iota_E) p (p \pi_H - \bar{u} - 1) \end{array} \right].$$

To see a separating equilibrium exists, first note that  $\mu_K(\iota_V | z, \tau_{\iota_V}) = 1$ . If  $\iota_V = \{G\}$ , then  $\mu_E(G | \{U\}, z, \tau_{\{G\}}) = 1$ , which implies  $\underline{R}_0^I(\tau_{\{G\}}) = \bar{R}_0^I$ . Thus, the investor optimally chooses  $R_0^I(\tau_{\{G\}}) = \bar{R}_0^I$  and receives a payoff of  $\bar{\Pi}(\lambda = 1, q = 1) - z$ . If  $\iota_V = \{U\}$ , then the uninformed entrepreneur believes with probability  $\mu_E(G | \{U\}, z, \tau_{\{G\}}) = \lambda$  that he holds a good project and  $\underline{R}_0^I(\tau_{\{U\}}) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ . The investor's expected payoff if he chooses  $\bar{R}_0^I$  is  $\bar{\Pi} - z$ , whereas his payoff if he chooses  $\underline{R}_0^I(\tau_{\{U\}})$  is  $\underline{\Pi} - z$ . This implies  $R_0^I(\tau_{\{U\}}) = \bar{R}_0^I$  since  $\bar{\Pi} > \underline{\Pi}$ . If  $\iota_V = \{B\}$ , then the investor's payoff is  $-z \forall R_0^I \in \mathbb{R}^+$ . The VC of type  $\{G\}$  has not incentive to deviate since he receives a payoff of  $p \bar{u}$  if he conforms, and

at most this amount if he deviates. If  $R_0^I(\tau_{\{B\}}) \leq \underline{R}_0^I(\tau_{\{U\}})$ , then the VC of type  $\{U\}$  has an incentive to deviate and report  $\tau_{\{B\}}$ . To see this note that he gets a payoff of  $\lambda p \bar{u}$  if he deviates compared to a payoff of  $\lambda q p \bar{u}$  if otherwise. Hence, to sustain such equilibria we need  $R_0^I(\tau_{\{B\}}) > \underline{R}_0^I(\tau_{\{U\}})$ . The investor's ex ante expected payoff is  $\bar{\Pi} + (1 - q) \bar{\Pi}$  and VCs optimally choose  $z = \min \{d - (1 - m_H), (1 - q) \bar{\Pi}\}$ .

For the semi-pooling equilibria in which  $S(\tau) = \{\{U\}, \{B\}\}$ , we have  $\mu_K(\{G\} | z, \tau_{\{G\}}) = 1$ ,  $\mu_K(\{U\} | z, \tau) = \frac{1-q}{1-q+q(1-\lambda)}$  and  $\mu_K(\{B\} | z, \tau) = \frac{q(1-\lambda)}{1-q+q(1-\lambda)}$ . If  $\iota_V = \{G\}$ , then the uninformed entrepreneur has belief  $\mu_E(G | \{U\}, z, \tau_{\{G\}}) = 1$  that he holds a good project, which implies  $\underline{R}_0^I(\tau_{\{G\}}) = \bar{R}_0^I$ . The investor chooses  $R_0^I(\tau_{\{G\}}) = \bar{R}_0^I$  and receives a payoff of  $\bar{\Pi}(\lambda = 1, q = 1) - z$ . Following report  $\tau$ , the uninformed entrepreneur has belief  $\mu_E(G | \{U\}, z, \tau) = \frac{(1-q)\lambda}{(1-q)+q(1-\lambda)}$  that his project is good, which implies  $\underline{R}_0^I(\tau) = \pi_H - \left(\frac{1-\lambda q}{1-q}\right) \frac{m_H + \bar{u}}{\lambda p}$ . Calculating expected payoffs, the investor receives a payoff of

$$\frac{1}{1-q+q(1-\lambda)} [(1-q) \lambda q \bar{\Pi}(\lambda = 1, q = 1)] - z.$$

if he chooses  $\bar{R}_0^I$  and a payoff of

$$\frac{1}{1-q+q(1-\lambda)} [(1-q) \lambda q \bar{\Pi}(\lambda = 1, q = 1)] + \underline{\Pi} - \bar{\Pi} - \frac{q(1-\lambda)}{1-q+q(1-\lambda)} (m_H + \bar{u}) - z,$$

if he chooses  $\underline{R}_0^I(\tau)$ . Therefore, the investor chooses  $R_0^I(\tau) = \bar{R}_0^I$  since  $\underline{\Pi} - \bar{\Pi} < 0$ . We need to check that no VC wants to deviate. The VC of type  $\{G\}$  receives a payoff the of  $p \bar{u}$  if he conforms with  $\tau_{\{G\}}$  and at most  $p \bar{u}$  if he deviates. Thus, he has no incentive to deviate. The VC of type  $\{B\}$  receives a payoff of 0 in any outcome and therefore has no incentive to deviate either. The VC of type  $\{U\}$  receives a payoff of  $q \lambda p \bar{u}$  if he conforms and a payoff

of  $\lambda p \bar{u}$  if he deviates to  $R_0^I(\tau')$ , provided that  $R_0^I(\tau') \leq \underline{R}_0^I(\tau)$  for  $\tau' \in T \setminus \{\tau, \tau_{\{G\}}\}$ . Hence,  $R_0^I(\tau') > \underline{R}_0^I(\tau)$  to sustain such equilibria. The investor ex ante payoff is  $\bar{\Pi} + (1 - q) \underline{\Pi}$ , and VCs optimally charge  $\min \{d - (1 - m_H), (1 - q) \bar{\Pi}\}$ .

For (ii), we have  $\mu_K(\{B\} | z, \tau_{\{B\}}) = 1$ ,  $\mu_K(\{U\} | z, \tau) = \frac{1-q}{1-q+\lambda q}$  and  $\mu_K(\{G\} | z, \tau) = \frac{\lambda q}{1-q+\lambda q}$ . If  $\iota_V = \{B\}$ , then  $\mu_E(G | \{U\}, z, \tau_{\{B\}}) = 0$  and the investor's payoff is  $d - z$  for all  $R_0^I \in \mathbb{R}^+$ . The uninformed entrepreneur has belief  $\mu_E(G | \{U\}, z, \tau) = \frac{\lambda}{1-q+\lambda q}$  that his holds a good project upon observing report  $\tau$ , which implies  $\underline{R}_0^I(\tau) = \pi_H - (1 - q + \lambda q) \frac{m_H + \bar{u}}{\lambda p}$ . If the investor chooses  $R_0^I = \bar{R}_0^I$  his expected payoff is

$$\frac{1}{1 - q + \lambda q} \lambda q \bar{\Pi} (\lambda = 1, q = 1) - z,$$

while if he chooses  $R_0^I = \underline{R}_0^I(\tau)$  his payoff is

$$\begin{aligned} & \frac{1}{1 - q + \lambda q} \lambda q \bar{\Pi} (\lambda = 1, q = 1) + (1 - q) \underline{\Pi} (q = 0) - \\ & (1 - q) \frac{[q(1 - \lambda)]^2}{1 - q(1 - \lambda)} (m_H + \bar{u}) - z. \end{aligned}$$

Without loss of generality, suppose  $\underline{\Pi}(q = 0) - \frac{[q(1 - \lambda)]^2}{1 - q(1 - \lambda)} (m_H + \bar{u}) = 0$ . The investor is indifferent between choosing  $\bar{R}_0^I$  or  $\underline{R}_0^I(\tau)$ . Suppose he chooses the latter; i.e.,  $R_0^I(\tau) = \underline{R}_0^I(\tau)$ . The VC of type  $\{G\}$  receives  $p \bar{u}$  and at most this amount if he deviates. Analogously, the VC of type  $\{U\}$  receives a payoff of  $\lambda p \bar{u}$  if he conforms and at most this quantity if he deviates. The VC of type  $\{B\}$  receives 0 in any outcome and has clearly no incentive to deviate. The investor's ex ante payoff in this type of equilibria is  $\bar{\Pi} + (1 - q) \underline{\Pi}$  and VCs optimally choose  $z = \min \{d - (1 - m_H), (1 - q) \underline{\Pi}\}$ . If the investor chooses the former, then the VCs of type  $\{G\}$  and  $\{U\}$  want to deviate provided that either  $R_0^I(\tau_{\{B\}}) \leq \underline{R}_0^I(\tau)$  or  $R_0^I(\tau') \leq \underline{R}_0^I(\tau)$ .

for  $\tau' \in T \setminus \{\tau, \tau_{\{B\}}\}$ . To see this, note that the VC of type  $\{G\}$  receives a payoff of  $qp\bar{u}$  and the VC of type  $\{U\}$  receives a payoff of  $\lambda qp\bar{u}$  if they conform. If they deviate and report either  $\tau_{\{B\}}$  or  $\tau'$  they receive payoffs of  $p\bar{u}$  and  $\lambda p\bar{u}$  respectively. Thus, we need both  $R_0^I(\tau_{\{B\}}) > \underline{R}_0^I(\tau)$  and  $R_0^I(\tau') > \underline{R}_0^I(\tau)$  to sustain such equilibria. However, this type of equilibria give the investor an ex ante expected payoff of  $\bar{\Pi}$ , and no VC-equilibria exist. If  $\underline{\Pi}(q=0) - \frac{[q(1-\lambda)]^2}{1-q(1-\lambda)}(m_H + \bar{u}) < 0$  then the investor must choose  $R_0^I(\tau) = \overline{R}_0^I$  and we already know a VC-equilibrium does not exist in this situation. Therefore, VC-equilibria in which  $S(\tau) = \{\{G\}, \{U\}\}$  exist if and only if  $\underline{\Pi}(q=0) - \frac{[q(1-\lambda)]^2}{1-q(1-\lambda)}(m_H + \bar{u}) \geq 0$ .

To show (iii), note that  $\mu_K(\{U\} | z, \tau_{\{B\}}) = 1$ ,  $\mu_K(\{G\} | z, \tau) = \lambda$  and  $\mu_K(\{B\} | z, \tau) = 1 - \lambda$ . If  $\iota_V = \{U\}$ , then  $\mu_E(G | \{U\}, z, \tau_{\{U\}}) = \lambda$ , which implies  $\underline{R}_0^I(\tau_{\{U\}}) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ . If the investor chooses  $\overline{R}_0^I$  he receives a payoff of  $\lambda q \bar{\Pi} (\lambda = 1, q = 1) - z$ . If he chooses  $\underline{R}_0^I(\tau_{\{U\}})$  his payoff is

$$\lambda q \bar{\Pi} (\lambda = 1, q = 1) + (1 - q) \underline{\Pi}(q = 0) - q(1 - \lambda)(m_H + \bar{u}) - z.$$

Therefore, he chooses  $\underline{R}_0^I(\tau_{\{U\}})$  only if  $(1 - q) \underline{\Pi}(q = 0) - q(1 - \lambda)(m_H + \bar{u}) \geq 0$ . After observing  $\tau$  the uninformed entrepreneur believes he holds a good project with probability  $\mu_E(G | \{U\}, z, \tau) = \lambda$ , which implies  $\underline{R}_0^I(\tau) = \pi_H - \frac{m_H + \bar{u}}{\lambda p}$ . Therefore, the expected investor's expected payoff is the same as that when he faces the VC of type  $\{U\}$ . This in turn implies that he chooses  $\underline{R}_0^I(\tau)$  only if  $(1 - q) \underline{\Pi}(q = 0) - q(1 - \lambda)(m_H + \bar{u}) \geq 0$ . If  $(1 - q) \underline{\Pi}(q = 0) - q(1 - \lambda)(m_H + \bar{u}) < 0$ , then the investor chooses  $R_0^I(\tau_U) = R_0^I(\tau) = \overline{R}_0^I$  and a VC-equilibrium, if it exists, gives the investor an ex ante expected payoff of  $\bar{\Pi}$ . But then  $e_K = 0$  and we have a contradiction, which implies a VC-equilibrium does not exist.



Suppose  $(1 - q) \underline{\Pi}(q = 0) - q(1 - \lambda)(m_H + \bar{u}) \geq 0$ . Without loss of generality, assume the investor chooses  $R_0^I(\tau_U) = R_0^I(\tau) = \underline{R}_0^I(\tau)$ . Clearly no VC has an incentive to deviate and then investor's implied ex ante expected payoff is  $\underline{\Pi}$ , which implies a VC-equilibrium does not exist.

**Proof of Proposition 3** Suppose they are strong announcement-proof. We claim that the announcement  $\langle D, N \rangle$  with  $N = D = \{\{G\}, \{U\}\}$  is a credible announcement relative these equilibria. If this announcement is believed, then the beliefs of investors and entrepreneurs are updated according to (12). In particular, the beliefs of the investor and uninformed entrepreneur are the same as in the proof of (ii) of Proposition 2:  $\hat{\mu}_K(\{U\} | z, D) = \frac{1-q}{1-q+\lambda q}$ ,  $\hat{\mu}_K(\{G\} | z, D) = \frac{1-q}{1-q+\lambda q}$ , and  $\mu_{E_1}(G | \{U\}, z, D) = \frac{\lambda}{1-q+\lambda q}$ . Therefore, the equilibria induced by these beliefs are the same as those in (ii) of Proposition 2 and this announcement is *credible* since: (1) the payoff of the VC of type  $\{G\}$  in (ii) is the same as his payoff in (i), (2) the payoff of the VC of type  $\{U\}$  in (ii) is strictly preferred to his payoff in (i), (3) the payoff of the VC of type  $\{B\}$  in (ii) is the same as his payoff in (i), and (4) there is clearly no other announcement that, if believed, could increase the payoffs of the VCs of types  $\{G\}$  and  $\{U\}$ . Hence, we have a contradiction. That the equilibria in (ii) of Proposition 2 are strongly announcement-proof follows from the observation that all VCs receive their maximum expected payoff in those equilibria.

**Proof of Proposition 4** We omit the proof of Proposition 4 since it is nearly identical to that of Proposition 2.

# Appendix B

## Refinement Criterion

We first state the payoffs of the VC and the investor of the cheap-talk game that starts when  $k = 1$ . The investor's expected payoff given his belief upon receiving report  $\tau$  is:

$$u_K(R_0^I, \tau) \equiv \sum_{\iota_E \in T} \mu_K(\iota_E | z, \tau) \left[ \begin{array}{l} e_{\iota_E}^I(R_0^I(\tau)) (\mu(G|\iota_E) p R_0^I(\tau) - (1 - m_H) - z) - \\ (1 - e_{\iota_E}^I(R_1(\tau))) z + e_{\iota_E}^I \mu(G|\iota_E) p (p\pi_H - \bar{u} - 1) \end{array} \right]. \quad (\text{B.1})$$

The expected payoff of the VC of type  $\iota_{E_2}$  is:

$$u_V(R_0^I, \tau_{\iota_V}, \iota_V) \equiv \sum_{\iota_E \in T} e_{\iota_E}^I(R_1(\tau_{\iota_V})) \mu(\iota_E | \iota_V) \mu(G|\iota_E, \iota_V) p \bar{u}. \quad (\text{B.2})$$

It is clear that the VC of type  $\{U\}$  prefers those equilibria in (ii) to those in (i). To see this note that his payoff in (i) is  $u_V(R_0^I, \tau_{\{U\}}, \{U\}) = \lambda q p \bar{u}$ , while his payoff in (ii) is  $u_V(R_0^I, \tau_{\{U\}}, \{U\}) = \lambda p \bar{u}$ . However, VCs of type  $\{G\}$  and  $\{B\}$  are indifferent between equilibria played in (i) and equilibria played in (ii) since in each of these equilibria they receive  $u_V(R_0^I, \tau_{\{G\}}, \{G\}) = p \bar{u}$  and  $u_V(R_0^I, \tau_{\{B\}}, \{B\}) = 0$  respectively.

The idea behind the “strongly announcement-proof” criterion of Matthews et al. is sim-

ple. Consider the equilibria in (i) and let  $N$  be a non-empty collection of non-empty and disjoint subsets of  $T$ . Suppose  $D = \{\{G\}, \{U\}\} \in N$  and the VC of type  $\{U\}$  announces: (1) his type is in  $D = \{\{G\}, \{U\}\}$ , (2) if his type was in another set in  $N$  he would have announced it, and (3) if his type was not in  $N$  he would have played his equilibrium strategy instead of making this announcement. If the investor and the entrepreneur believe this announcement, they update their beliefs so that:

$$\hat{\mu}_K(\iota_V|z, D) = \begin{cases} \frac{\mu_K(\iota_V|z)}{\sum_{\iota'_V \in D} \mu(\iota'_V|z)} & \text{if } \iota_V \in D \\ 0 & \text{if } \iota_V \notin D \end{cases}, \quad (\text{B.3})$$

$$\hat{\mu}_E(\iota_V|\iota_E, z, D) = \begin{cases} \frac{\mu_E(\iota_V|\iota_E, z)}{\sum_{\iota'_V \in D} \mu(\iota'_V|\iota_E, z)} & \text{if } \iota_V \in D \\ 0 & \text{if } \iota_V \notin D \end{cases}. \quad (\text{B.4})$$

Formally, an announcement is a pair  $\langle D, N \rangle$  where  $N$  is an announcement strategy and  $D \in N$ . The set of deviant types is  $T(N) = \{\iota_V \in T : \exists D \in N : \iota_V \in D\}$ . Let  $u_V(R_0^I, \tau_{\iota_V}, \iota_V)$  be the payoff of the VC of type  $\iota_V$  in some equilibrium in  $A(\bar{\Pi})$ ,  $\underline{u}_V(R_0^I, D, \iota_V)$  be the minimum payoff that VC of type  $\iota_V$  receives in an equilibrium if announcement  $\langle D, N \rangle$  is believed, and let  $\bar{u}_V(R_0^I, D, \iota_V)$  be the maximum payoff the VC of type  $\iota_V$  receives if announcement  $\langle D, N \rangle$  is believed.

**Definition 1** *An announcement strategy  $N$  and the corresponding announcements  $\langle D, N \rangle$  are weakly credible relative to an equilibrium in  $A(\bar{\Pi})$  if:*

(i)  $\underline{u}_V(R_0^I, D, \iota_V) \geq u_V(R_0^I, \tau_{\iota_V}, \iota_V)$  for all  $D \in N$  and  $\iota_V \in D$  with at least one strict inequality;

(ii)  $\bar{u}_V(R_0^I, D, \iota_V) \leq u_V(R_0^I, \tau_{\iota_V}, \iota_V)$  for all  $\iota_V \in T/T(N)$

(iii)  $\underline{u}_V(R_0^I, D, \iota_V) \geq \underline{u}_V(R_0^I, \bar{D}, \iota_V)$  for all  $\bar{D}, D \in N$  and  $\iota_V \in D$ .

*If no announcement is weakly credible, then this equilibrium and its outcome are strongly announcement-proof.*

# Appendix C

## Proofs of Chapter 2

**Proof of Proposition 3** Suppose  $R_1 > y_2$ . In this case borrower always triggers renegotiation. The lender's payoff is  $x(L(\pi)) - f$  if he buys a CDS with  $\pi \leq \pi^*$  and  $L(\pi) - f$  if he buys a CDS with  $\pi > \pi^*$ . In the former case, since a credit event never occurs and the CDS provider is competitive,  $f = 0$ . The lender's payoff is maximized when he chooses  $\pi = \pi^*$ , which yields him a payoff of  $\delta\lambda y_2$ . In the latter case, the competitive CDS provider charges  $f = \pi$  and the lender's payoff is  $\beta I$  regardless of his CDS position. Therefore, since  $\delta\lambda y_2 > \beta I$ , the lender buys a CDS with  $\pi = \pi^*$ . Suppose  $R_1 \in (\delta y_2, y_2]$ . If the lender buys a CDS with  $\pi \leq \pi^*$ , then

$$R_1 > \delta y_2 = \delta(1 - \lambda)y_2 + \delta\lambda y_2 = \delta(1 - \lambda)y_2 + x(L(\pi^*)) \geq \delta(1 - \lambda)y_2 + x(L(\pi)).$$

Therefore, the borrower always triggers renegotiation. The lender's payoff is  $\delta\lambda y_2 - f$ . Because a credit event never occurs, the competitive CDS provider charges  $f = 0$ . Therefore, the lender's payoff is  $\delta\lambda y_2$ . If the lender buys a CDS with  $\pi > \pi^*$ , then since  $R_1 \leq y_2$ , the

borrower does not trigger strategic renegotiation. The lender's payoff is

$$\mu [p_H R_1 + (1 - p_H) (\beta I + \pi)] + (1 - \mu) [p_L R_1 + (1 - p_L) (\beta I + \pi)] - f.$$

Since the breakeven condition for the competitive CDS provider is

$$f = \pi [\mu (1 - p_H) + (1 - \mu) (1 - p_L)],$$

the lender's payoff is  $\mu [p_H R_1 + (1 - p_H) \beta I] + (1 - \mu) [p_L R_1 + (1 - p_L) \beta I]$ . Therefore, the lender buys a CDS with  $\pi > \pi^*$  if and only if

$$\begin{aligned} \mu [p_H R_1 + (1 - p_H) \beta I] + (1 - \mu) [p_L R_1 + (1 - p_L) \beta I] &> \delta \lambda y_2 \iff \\ R_1 &> \frac{\delta \lambda y_2 - \beta I [\mu (1 - p_H) + (1 - \mu) (1 - p_L)]}{\mu p_H + (1 - \mu) p_L}. \end{aligned}$$

Suppose  $R_1 \leq \delta y_2$ . In this case the borrower never calls for strategic renegotiation. In

the lender buys a CDS with  $\pi \leq \pi^*$ , his payoff is

$$\mu \{p_H R_1 + (1 - p_H) x(L(\pi))\} + (1 - \mu) \{p_L R_1 + (1 - p_L) x(L(\pi))\} - f.$$

Since a credit event never occurs, the competitive CDS provider charges  $f = 0$ . Since the lender's payoff is increasing in  $\pi$ , it is optimal to choose  $\pi = \pi^*$ . Therefore, the lender's payoff is

$$\mu \{p_H R_1 + (1 - p_H) \delta \lambda y_2\} + (1 - \mu) \{p_L R_1 + (1 - p_L) \delta \lambda y_2\}.$$

If the lender buys a CDS with  $\pi > \pi^*$ , then his payoff is

$$\mu \{p_H R_1 + (1 - p_H) (\beta I + \pi)\} + (1 - \mu) \{p_L R_1 + (1 - p_L) (\beta I + \pi)\} - f.$$

Because the competitive CDS provider charges  $f = \pi [\mu (1 - p_H) + (1 - \mu) (1 - p_L)]$ , the lender's payoff is  $\mu \{p_H R_1 + (1 - p_H) \beta I\} + (1 - \mu) \{p_L R_1 + (1 - p_L) \beta I\}$ . Clearly, it is optimal for the lender to demand  $\pi = \pi^*$ .

**Proof of Proposition 4** For  $R_1 > y_2$ , the borrower's payoff if he chooses high effort is  $p_H (y_1 + (1 - \delta) y_2) + (1 - p_H) (1 - \delta) y_2$  and his payoff if he chooses low effort is given by  $p_L (y_1 + (1 - \delta) y_2) + (1 - p_L) (1 - \delta) y_2 + B$ . Therefore, the borrower chooses high effort if and only if  $y_1 (p_H - p_L) \geq B$ . Suppose  $R_1 \in (\delta y_2, y_2]$ . If the lender has a CDS with  $\pi > \pi^*$ , the borrower's payoff if he chooses high effort is  $p_H (y_1 - R_1 + y_2)$  and if he chooses low effort is  $p_L (y_1 - R_1 + y_2) + B$ . Therefore, the borrower chooses high effort if and only if  $y_1 + y_2 - \frac{B}{p_H - p_L} \geq R_1$ . If the lender has a CDS with  $\pi \leq \pi^*$ , the borrower's payoff if he chooses high effort is

$$p_H [y_1 + (1 - \delta) y_2] + (1 - p_H) (1 - \delta) y_2.$$

The borrower's payoff if he chooses low effort is

$$p_L [y_1 + (1 - \delta) y_2] + (1 - p_L) (1 - \delta) y_2 + B.$$

Therefore, the borrower chooses high effort if and only if  $y_1 (p_H - p_L) \geq B$ . Suppose  $R_1 \leq \delta y_2$ . The borrower's payoff if he chooses high effort is

$$p_H [y_1 - R_1 + y_2] + (1 - p_H) (1 - \delta) y_2.$$

The borrower's payoff if he chooses low effort is

$$p_L [y_1 - R_1 + y_2] + (1 - p_L) (1 - \delta) y_2 + B.$$

Therefore, the borrower chooses high effort if and only if

$$y_1 + \delta y_2 - \frac{B}{p_H - p_L} \geq R_1.$$

**Proof of Proposition 5** One must note that (1) and (3) follow directly from Propositions 4 and 5. For (2), let  $R_1 \in (R(0), y_2]$ . One must note that  $R_1 > R(0) \geq R(\mu)$ , which implies that the lender buys a CDS with  $\pi > \pi^*$ . If  $R_1 > \overline{R_1}$ , then the borrower chooses  $e_L$ , which implies  $\mu = 0$ . Therefore  $(\pi > \pi^*, e = e_L, \mu = 0)$  is the unique equilibrium of the CDS–Effort game, which establishes (a). If  $R_1 \leq \overline{R_1}$ , then the borrower chooses  $e_H$  and we have  $\mu = 1$ . As a result,  $R_1 > R(0) \geq R(1)$  and  $(\pi > \pi^*, e = e_H, \mu = 1)$  is the unique equilibrium of the CDS–Effort game, which establishes (b).

Let  $R_1 \in (\delta y_2, R(0)]$ . Suppose  $R_1 > \overline{R_1}$ . If  $\mu$  is such that  $R_1 > R(\mu)$ , the lender chooses  $\pi > \pi^*$ . The borrower chooses  $e_L$ , which implies  $\mu = 0$ . But then we have a contradiction since  $R_1 \leq R(0)$  and choosing  $\pi = \pi^*$  is optimal for the lender. If  $\mu$  is such that  $R_1 \leq R(\mu)$ , the lender chooses  $\pi = \pi^*$ . The borrower chooses  $e_L$ , which implies  $\mu = 0$ . Therefore,  $R_1 \leq R(\mu) \leq R(0)$  and  $(\pi = \pi^*, e = e_L, \mu = 0)$  is the unique equilibrium of the CDS–Effort game.

Suppose  $R_1 \leq \overline{R_1}$ . If  $\mu$  is such that  $R_1 > R(\mu)$ , the lender chooses  $\pi > \pi^*$ . The borrower chooses  $e_H$ , which implies  $\mu = 1$ . Therefore,  $R_1 > R(\mu) \geq R(1)$  and  $(\pi > \pi^*, e = e_H, \mu = 1)$  is an equilibrium. If  $\mu$  is such that  $R_1 \leq R(\mu)$ , the lender chooses  $\pi = \pi^*$ . As a consequence, the borrower chooses  $e_L$ , which implies  $\mu = 0$ . Therefore,  $R_1 \leq R(\mu) \leq R(0)$  and  $(\pi = \pi^*, e = e_L, \mu = 0)$  is an equilibrium.

**Proof of Proposition 6** Suppose  $R_1$  is such that  $R_1 \leq \delta y_2$ . Without loss of generality, let  $\underline{R_1} \in [0, \delta y_2]$ . If the lender chooses  $R_1 = \underline{R_1}$ , then the borrower chooses  $e_H$ , the lender buys a

CDS with  $\pi = \pi^*$ , and consistency of beliefs implies  $\mu = 1$ . We know that the borrower does not call for strategic renegotiation. The lender's expected payoff is  $p_H \underline{R}_1 + (1 - p_H) \delta \lambda y_2$ . If the lender chooses  $\delta y_2 = R_1$ , the borrower chooses  $e_L$ , the lender buys a CDS with  $\pi = \pi^*$ , and consistency of beliefs implies  $\mu = 0$ . We know that the borrower does not call for strategic renegotiation. The lender's expected payoff is  $p_L R_1 + (1 - p_L) \delta \lambda y_2$ . The lender chooses  $R_1 = \underline{R}_1$  if and only if  $y_2 \geq \underline{y}_2 \equiv -\frac{p_H}{\delta(p_H - p_L)(1 - \lambda)} \Delta$ .

Suppose he chooses  $R_1 \in (\delta y_2, y_2]$ . If the lender chooses  $R_1 = \overline{R}_1$ , then the borrower chooses  $e_H$ , the lender buys a CDS with  $\pi > \pi^*$ , and consistency of beliefs implies  $\mu = 1$ . The borrower does not trigger strategic renegotiation. The lender's expected payoff is  $p_H \overline{R}_1 + (1 - p_H) \beta I$ . If the lender chooses  $R_1 = y_2$ , then the borrower chooses  $e_L$ , the lender buys a CDS with  $\pi > \pi^*$ , and consistency of beliefs implies  $\mu = 0$ . The borrower does not call for strategic renegotiation. The lender expected payoff is  $p_L y_2 + (1 - p_L) \beta I$ . The lender chooses  $R_1 = \overline{R}_1$  if and only if  $p_H \overline{R}_1 + (1 - p_H) \beta I \geq p_L y_2 + (1 - p_L) \beta I \iff y_2 \geq \underline{y}_2 \equiv -\frac{p_H}{p_H - p_L} \Delta + \beta I$ . If  $R_1 > y_2$  then the borrower chooses  $e_L$ , the lender buys a CDS with  $\pi = \pi^*$ , and consistency of beliefs implies  $\mu = 0$ . The borrower triggers strategic renegotiation. The lender's payoff is  $\delta \lambda y_2$ .

**Proof of Proposition 7**  $\max \{ \Pi(y_2), \Pi(\overline{R}_1) \} = \Pi(y_2)$ ,  $\max \{ \Pi(\delta y_2), \Pi(\underline{R}_1) \} = \Pi(\delta y_2)$  for  $y_2 < \underline{y}_2$ . Since  $\Pi(y_2) > \Pi(\delta y_2)$  for  $y_2$  small, there exists  $y_2^*$  such that  $\Pi(y_2^*) = \Pi(\delta y_2^*)$  if and only if the slope of  $\Pi(\delta y_2)$  is higher than that of  $\Pi(y_2)$ . This is true if and only if  $c(p_L) > p_L$ . In this case, direct calculations show that  $y_2^* < \underline{y}_2$  if and only if the liquidation value is sufficiently small, i.e.,  $\beta I < \frac{-p_H \Delta [c(p_L) - p_L]}{(p_H - p_L)[1 - c(p_L)]}$ . We also have that  $\max \{ \Pi(y_2), \Pi(\overline{R}_1) \} = \Pi(\overline{R}_1)$  and  $\max \{ \Pi(\delta y_2), \Pi(\underline{R}_1) \} = \Pi(\delta y_2)$  for  $\underline{y}_2 \leq y_2 < \overline{y}_2$ . Since  $\Pi(\delta y_2) > \Pi(\overline{R}_1)$  for  $y_2$



small, there exists  $y_2^{**}$  such that  $\Pi(\delta y_2^{**}) = \Pi(\overline{R_1})$  if and only if the slope of  $\Pi(\overline{R_1})$  is higher than that of  $\Pi(\delta y_2)$ . This is true if and only if  $p_H > c(p_L)$ . Finally, we have that  $\max\{\Pi(y_2), \Pi(\overline{R_1})\} = \Pi(\overline{R_1})$  and  $\max\{\Pi(\delta y_2), \Pi(\underline{R_1})\} = \Pi(\underline{R_1})$  for  $y_2 \geq \overline{y_2}$ . Since  $\Pi(\overline{R_1}) > \Pi(\underline{R_1})$  for  $y_2$  small, there exists  $y_2^{***}$  such that  $\Pi(\overline{R_1}) = \Pi(\underline{R_1})$  if and only if the slope of  $\Pi(\underline{R_1})$  is higher than that of  $\Pi(\overline{R_1})$ . This is true if and only if  $c(p_H) > p_H$ .

**Case 1** ( $p_H > p_L \geq c(p_H) > c(p_L)$ ): In this case, the lender chooses  $R_1 = y_2$  for  $y_2 < \underline{y_2}$  and  $R_1 = \overline{R_1}$  for  $y_2 \geq \overline{y_2}$ . It must follow that  $R_1 = \overline{R_1}$  for  $\underline{y_2} \leq y_2 < \overline{y_2}$ . To see this, suppose otherwise, i.e.,  $y_2^{**}$  is such that either  $y_2^{**} \geq \overline{y_2}$  or  $\underline{y_2} < y_2^{**} < \overline{y_2}$ . If the latter holds then, because  $\Pi(y_2) > \Pi(\delta y_2)$  for all  $y_2$ , we have  $\Pi(y_2) > \Pi(\delta y_2) > \Pi(\overline{R_1})$  for  $\underline{y_2} < y_2 < y_2^{**}$ . But this contradicts the definition of  $\underline{y_2}$ . If the former holds then, because  $\Pi(\overline{R_1}) > \Pi(\underline{R_1})$  for all  $y_2$ , we have  $\Pi(\delta y_2) \geq \Pi(\overline{R_1}) > \Pi(\underline{R_1})$  for  $\overline{y_2} \leq y_2 \leq y_2^{**}$ . This contradicts the definition of  $\overline{y_2}$ .

**Case 2** ( $p_H \geq c(p_H) > p_L \geq c(p_L)$ ): The analysis is the same as in case Case 1.

**Case 3** ( $p_H \geq c(p_H) > c(p_L) > p_L$ ): In this case, the lender chooses  $R_1 = \overline{R_1}$  for  $y_2 \geq \overline{y_2}$ . There are two cases to consider: (1)  $y_2^* \geq \underline{y_2}$  and (2)  $y_2^* < \underline{y_2}$ . If (1) holds, then we are back to Cases 1 and 2 as  $R_1 = y_2$  for  $y_2 < \underline{y_2}$  and  $R_1 = \overline{R_1}$  for  $\underline{y_2} \leq y_2 < \overline{y_2}$ . The former follows by assumption since  $\Pi(y_2) > \Pi(\delta y_2)$  for all  $y_2$  such that  $y_2 < \underline{y_2}$ , which establishes. For the latter, suppose otherwise, i.e.,  $y_2^{**} > \underline{y_2}$ . First, let  $\underline{y_2} < y_2^{**} < \overline{y_2}$ . If  $y_2^*$  is such that  $\underline{y_2} \leq y_2^* < y_2^{**}$ , then we have  $\Pi(y_2) \geq \Pi(\delta y_2) > \Pi(\overline{R_1})$  for  $\underline{y_2} \leq y_2 \leq y_2^*$ , which contradicts the definition of  $\underline{y_2}$ . If  $y_2^* \geq y_2^{**}$ , then we have  $\Pi(y_2) > \Pi(\delta y_2) > \Pi(\overline{R_1})$  for  $\underline{y_2} < y_2 < y_2^{**}$ , which also contradicts the definition of  $\underline{y_2}$ . Second, let  $y_2^{**} \geq \overline{y_2}$ . In this case, because  $\Pi(y_2) > \Pi(\delta y_2)$  for all  $y_2$ , we have  $\Pi(\delta y_2) \geq \Pi(\overline{R_1}) > \Pi(\underline{R_1})$  for  $\overline{y_2} \leq y_2 \leq y_2^{**}$ , which contradicts the definition of  $\overline{y_2}$ .

If (2) holds, then  $R_1 = y_2$  for  $y_2 < y_2^*$  and  $R_1 = \delta y_2$  for  $y_2^* \leq y_2 < \underline{y}_2$ . This implies  $\underline{y}_2 < y_2^{**} < \overline{y}_2$  such that  $R_1 = \delta y_2$  for  $\underline{y}_2 \leq y_2 < y_2^{**}$  and  $R_1 = \overline{R}_1$  for  $y_2^{**} \leq y_2 < \overline{y}_2$ . To see this, suppose otherwise. i.e., either  $y_2^{**} \geq \overline{y}_2$  or  $y_2^{**} < \underline{y}_2$ . If  $y_2^{**} \geq \overline{y}_2$  then, since  $\Pi(\overline{R}_1) > \Pi(\underline{R}_1)$  for all  $y_2$ , we have that  $\Pi(\delta y_2) \geq \Pi(\overline{R}_1) > \Pi(\underline{R}_1)$  for  $\overline{y}_2 \leq y_2 \leq y_2^{**}$ , which contradicts the definition of  $\overline{y}_2$ . If  $y_2^{**} < \underline{y}_2$ , then we have that  $\Pi(\overline{R}_1) > \Pi(\delta y_2) > \Pi(y_2)$  for  $y_2^{**} < y_2 < \underline{y}_2$ , which contradicts the definition of  $\underline{y}_2$ .

**Case 4** ( $c(p_H) > c(p_L) \geq p_H > p_L$ ): In this case, the lender chooses  $R_1 = \delta y_2$  for  $\underline{y}_2 \leq y_2 < \overline{y}_2$ . This implies that  $y_2^{***} \leq \overline{y}_2$ . To see this, suppose otherwise, i.e.,  $y_2^{***} > \overline{y}_2$ . Since  $\Pi(\delta y_2) > \Pi(\overline{R}_1)$  for all  $y_2$ , we have that  $\Pi(\delta y_2) > \Pi(\overline{R}_1) > \Pi(\underline{R}_1)$  for  $\overline{y}_2 \leq y_2 < y_2^{***}$ . But this contradicts the definition of  $\overline{y}_2$ . Therefore, it follows that  $y_2^{***} \leq \overline{y}_2$ , which implies that  $R_1 = \underline{R}_1$  for  $y_2 \geq \overline{y}_2$ . Finally, it must be that  $y_2^* < \underline{y}_2$ . To see this, suppose otherwise, i.e.,  $y_2^* \geq \underline{y}_2$ . If  $y_2^* < \overline{y}_2$ , then we have that  $\Pi(y_2) \geq \Pi(\delta y_2) > \Pi(\overline{R}_1)$  for  $\underline{y}_2 \leq y_2 \leq y_2^*$ , which contradicts the definition of  $\underline{y}_2$ . If  $y_2^* \geq \overline{y}_2$ , then  $\Pi(y_2) \geq \Pi(\delta y_2) > \Pi(\overline{R}_1)$  for  $\underline{y}_2 \leq y_2 < \overline{y}_2$ , which also contradicts the definition of  $\underline{y}_2$ .

**Case 5** ( $c(p_H) > p_H > p_L \geq c(p_L)$ ): This case is impossible. To see this note that, by the definition of  $c(p)$ ,  $p \geq c(p)$  if and only if  $p \geq \delta\lambda + \delta(1-\lambda)p$ , which is true if and only if  $p \geq \frac{\delta\lambda}{1-\delta(1-\lambda)}$ . But this implies that  $p_L \geq \frac{\delta\lambda}{1-\delta(1-\lambda)} > p_H$ , which contradicts the assumption that  $p_H > p_L$ .

**Case 6** ( $c(p_H) > p_H > c(p_L) > p_L$ ): There are two cases to consider: (1)  $y_2^* \geq \underline{y}_2$  and (2)  $y_2^* < \underline{y}_2$ . If (1) holds, then from Case 3 we know that  $R_1 = y_2$  for  $y_2 < \underline{y}_2$  and  $R_1 = \overline{R}_1$  for  $\underline{y}_2 \leq y_2 < \overline{y}_2$ . It also follows that  $y_2^{***} \geq \overline{y}_2$ . To see this, suppose otherwise, i.e.,  $y_2^{***} < \overline{y}_2$ . If  $y_2^{***} \geq \underline{y}_2$ , then we have that  $\Pi(\underline{R}_1) \geq \Pi(\overline{R}_1) > \Pi(\delta y_2)$  for  $y_2^{***} \leq y_2 < \overline{y}_2$ ,

which contradicts the definition of  $\overline{y_2}$ . If  $y_2^{***} < \underline{y_2}$ , then  $\Pi(\underline{R_1}) \geq \Pi(\overline{R_1}) > \Pi(\delta y_2)$  for  $\underline{y_2} \leq y_2 < \overline{y_2}$ , which also contradicts the definition of  $\underline{y_2}$ .

If Case 2 holds, then from Case 3 we know that  $R_1 = y_2$  for  $y_2 < y_2^*$  and  $R_1 = \delta y_2$  for  $y_2^* \leq y_2 < \underline{y_2}$ . If  $y_2^{**} \geq \overline{y_2}$ , then the lender chooses  $R_1 = \delta y_2$  for  $\underline{y_2} \leq y_2 < \overline{y_2}$ . This implies that we are back to Case 4, from which it follows that  $y_2^{***} \leq \overline{y_2}$  and the lender chooses  $R_1 = \underline{R_1}$  for  $y_2 \geq \overline{y_2}$ . If  $y_2^{**} < \overline{y_2}$ , then it must be that  $\underline{y_2} < y_2^{**} < \overline{y_2}$ . To see this, suppose otherwise, i.e.,  $y_2^{**} \leq \underline{y_2}$ . For  $y_2^{**} > y_2^*$ , it follows that  $\Pi(\overline{R_1}) \geq \Pi(\delta y_2) > \Pi(y_2)$  for  $y_2^{**} \leq y_2 \leq \underline{y_2}$ , which contradicts the definition of  $\underline{y_2}$ . For  $y_2^{**} \leq y_2^*$ , it follows that  $\Pi(\overline{R_1}) > \Pi(\delta y_2) > \Pi(y_2)$  for  $y_2^* < y_2 < \underline{y_2}$ , which also contradicts the definition of  $\underline{y_2}$ . Therefore, the lender chooses  $R_1 = \delta y_2$  for  $\underline{y_2} \leq y_2 < y_2^{**}$  and  $R_1 = \overline{R_1}$  for  $y_2^{**} \leq y_2 < \overline{y_2}$ . Finally, it follows that  $y_2^{***} \geq \overline{y_2}$ . To see this suppose otherwise, i.e.,  $y_2^{***} < \overline{y_2}$ . If  $y_2^{***} \geq y_2^{**}$ , then we have that  $\Pi(\underline{R_1}) > \Pi(\overline{R_1}) > \Pi(\delta y_2)$  for  $y_2^{***} < y_2 < \overline{y_2}$ , which contradicts the definition of  $\overline{y_2}$ . If  $y_2^{***} < y_2^{**}$ , then we have that  $\Pi(\underline{R_1}) > \Pi(\overline{R_1}) \geq \Pi(\delta y_2)$  for  $y_2^{**} \leq y_2 < \overline{y_2}$ , which also contradicts the definition of  $\overline{y_2}$ .

# Appendix D

## Proofs of Chapter 3

**Proof of Proposition 1** The proof follows from Proposition 1 in Steiner and Sakovics (2010), which characterizes the unique Bayesian Nash equilibrium for a general class of global games with heterogeneous agents, including mine.

**Proof of Proposition 2** This result follows from Propositions 2 and 4 in Steiner and Sakovics (2010). Their result relies on the same assumption used to prove the result in Proposition 1 with the additional assumption that  $B_g$  and  $-C_g$  are Lipschitz continuous in  $\theta$ , which is clearly satisfied in my setup since  $B_g$  and  $-C_g$  do not depend on  $\theta$ .

**Proof of Proposition 3** Without CDSs, the incremental payoff of lenders of group  $g$  of financing over not financing is  $U_g(r_g) = (1 - \lambda)r_g - \lambda L(a) - 1$ . The first order necessary

conditions are

$$-(1-\lambda)\beta p_R + \mu_1 p_R(1-\lambda) = 0, \quad (\text{D.1})$$

$$-(1-\lambda)(\alpha - \beta p_R) + \mu_2 \left( \frac{\alpha - \beta p_R}{1-\beta} \right) (1-\lambda) = 0, \quad (\text{D.2})$$

$$-(1-\lambda)\beta(r_R - r_S) + \mu_1 U_R(r_R) - \mu_2 \frac{\beta}{1-\beta} U_S(r_S) = 0, \quad (\text{D.3})$$

$$\mu_2 \frac{\alpha - \beta p_R}{1-\beta} U_S(r_S) = 0, \quad (\text{D.4})$$

$$\mu_1 p_R U_R(r_R) = 0, \quad (\text{D.5})$$

$$\mu_1, \mu_2 \geq 0. \quad (\text{D.6})$$

There are three cases to consider:  $p_R = 0$ ,  $p_R = \frac{\alpha}{\beta}$ , and  $0 < p_R < \frac{p_R}{\beta}$ . If  $p_R = 0$ , then D.2 implies  $\mu_2 > 0$  and D.4 implies that  $U_S(r_S) = 0 \Rightarrow r_S = \frac{1+\lambda L(a)}{1-\lambda}$ . If  $p_R = \frac{\alpha}{\beta}$ , then D.1 implies  $U_R(r_R) = 0 \Rightarrow r_R = \frac{1+\lambda L(a)}{1-\lambda}$ . If  $0 < p_R < \frac{p_R}{\beta}$ , then D.1 and D.2 imply  $\mu_1, \mu_2 > 0$ . Therefore, D.4 and D.5 imply  $U_S(r_S) = 0 \Rightarrow r_S = \frac{1+\lambda L(a)}{1-\lambda}$  and  $U_R(r_R) = 0 \Rightarrow r_R = \frac{1+\lambda L(a)}{1-\lambda}$ . All three cases yield the same payoff to the borrower, which leads us to conclude that the optimal repayment and financing probabilities are given by  $r^* = \frac{1+\lambda L(a)}{1-\lambda}$  and  $(p_R^*, p_S^*) = \left( p_R, \frac{\alpha - \beta p_R}{1-\beta} \right)$  for  $p_R \in \left[ 0, \frac{\alpha}{\beta} \right]$ .

**Proof of Proposition 4** Let the function  $Q : \{0, 1\} \times [0, 1] \rightarrow \mathbb{R}$  be such that  $Q(1, \lambda) = W^{**}(\lambda)$  and  $Q(0, \lambda) = W^*(\lambda)$ . This function has increasing differences in  $(\gamma, \lambda)$  if, for  $\lambda' > \lambda$ , it holds that

$$Q(1, \lambda') - Q(0, \lambda') \geq Q(1, \lambda) - Q(0, \lambda),$$

which is equivalent to  $W^{**}(\lambda) - W^*(\lambda)$  being increasing in  $\lambda$ . From Milgrom and Shannon (1994) it is known that, if  $Q(\gamma, \lambda)$  has increasing differences in  $(\gamma, \lambda)$ , then  $\arg \max_{\gamma \in \{0, 1\}} Q(\gamma, \lambda)$

is monotone non-decreasing in  $\lambda$ .

Let  $\bar{r}^{**}$  and  $\bar{r}^*$  be the average optimal repayment with and without CDSs, respectively.

Analogously, let  $\mu_R^{**}, \mu_S^{**}$  and  $\mu_R^*, \mu_S^*$  be the Lagrange multipliers with and without CDSs for the constraints of risky and safe lenders. Let  $H \equiv H(\bar{r}^{**}(1 - \phi^*))$  and  $h \equiv h(\bar{r}^{**}(1 - \phi^*))$ .

From the envelope theorem, at the optimum it holds that

$$\begin{aligned}\frac{\partial W^{**}(\lambda)}{\partial \lambda} &= -(\bar{y} - \bar{r}^{**}) + \int_{\bar{r}^{**}(1-\phi^*)}^{\infty} \theta h(\theta) d\theta - \mu_R^{**} p_R A_R^{**}(\lambda) - \mu_S^{**} \left( \frac{\alpha - \beta p_R}{1 - \beta} \right) A_S^{**}(\lambda), \\ \frac{\partial W^*(\lambda)}{\partial \lambda} &= -(\bar{y} - \bar{r}^*) + \int_0^{\infty} \theta h(\theta) d\theta - \mu_R^* p_R [r_R^* + L(a)] - \mu_S^* \left( \frac{\alpha - \beta p_R}{1 - \beta} \right) [r_S^* + L(a)],\end{aligned}$$

where

$$A_g^{**}(\lambda) = r_g^{**} + \left[ H L_g \left( \frac{a}{\lambda} \right) + (1 - H) L(a) \right] + \lambda \left\{ h \bar{r}^{**} \frac{\partial \phi^*}{\partial \lambda} \left[ L(a) - L_g \left( \frac{a}{\lambda} \right) \right] - H L'_g \left( \frac{a}{\lambda} \right) \frac{a}{\lambda^2} \right\}.$$

One can rewrite the first other conditions to show that  $\mu_g^{**} \geq \mu_g^*$ . Since  $\bar{r}^{**} \leq \bar{r}^*$  and  $\phi^* = 1$  if  $\lambda = 1$ , it follows that  $\frac{\partial W^{**}(1)}{\partial \lambda} - \frac{\partial W^*(1)}{\partial \lambda} \leq 0$ . Since  $\phi^* = 1$  and  $\bar{r}^{**} = \bar{r}^*$  if  $\lambda = 0$ , it follows that  $\frac{\partial W^{**}(0)}{\partial \lambda} - \frac{\partial W^*(0)}{\partial \lambda} \geq 0$ . Because  $\frac{\partial W^{**}(\lambda)}{\partial \lambda}$  and  $\frac{\partial W^*(\lambda)}{\partial \lambda}$  are continuous, there exists  $\underline{\lambda} \in [0, 1]$  such that  $\frac{\partial W^{**}(\underline{\lambda})}{\partial \lambda} = \frac{\partial W^*(\underline{\lambda})}{\partial \lambda}$ . In addition, because  $\frac{\partial W^{**}(\lambda)}{\partial \lambda}$  is decreasing in  $\lambda$ , for  $\lambda > \underline{\lambda}$ ,  $\frac{\partial W^{**}(\lambda)}{\partial \lambda} < \frac{\partial W^*(\lambda)}{\partial \lambda}$  and for  $\lambda < \underline{\lambda}$ ,  $\frac{\partial W^{**}(\lambda)}{\partial \lambda} > \frac{\partial W^*(\lambda)}{\partial \lambda}$ . Therefore,  $Q(\gamma, \lambda)$  has increasing differences in  $(\gamma, \lambda)$  for  $\lambda < \underline{\lambda}$  and decreasing if  $\lambda > \underline{\lambda}$ , which concludes the proof.

**Proof of Proposition 5** The borrower's problem is to maximize

$$Q(r_R, r_S, p_R, \lambda) = (1 - \lambda)(\bar{y} - p_R \beta r_R - (\alpha - \beta p_R) r_S) + \lambda \int_{[p_R \beta r_R + p_S(1-\beta)r_S](1-\phi^*)}^{\infty} \theta h(\theta) d\theta,$$

subject to a constraint set, which I call  $S$ . From Milgrom and Shannon (1994) we know that

$\arg \max_{x \in X} f(x, t)$  is monotone non-decreasing in  $t$  if and only if  $f$  is quasisupermodular in  $x$  and satisfies the single crossing property in  $(x; t)$ . A sufficient condition for a function to be quasisupermodular is for it to be supermodular. A twice continuously differentiable function  $f$  is supermodular in  $x$  if and only if  $\frac{\partial^2 f}{\partial x_i \partial x_j}$  for  $i \neq j$ .

It is easy to see that  $\bar{y} - p_R \beta r_R - (\alpha - \beta p_R) r_S$  is supermodular in  $(-r_R, r_S, p_R)$ . One can also check that  $[p_R \beta r_R + p_S (1 - \beta) r_S] (1 - \phi^*)$  is supermodular in  $(-r_R, r_S, p_R)$ . Since a monotonic transformation of a quasisupermodular function is quasisupermodular, it follows that

$$\int_{[p_R \beta r_R + p_S (1 - \beta) r_S] (1 - \phi^*)}^{\infty} \theta h(\theta)$$

is quasisupermodular. For the same reason, because  $Q$  is the sum of two positive quasisupermodular functions, it is also quasisupermodular. Finally, one can show that  $Q$  has increasing differences in  $(-r_R, r_S, p_R; -\lambda)$ , which implies  $Q$  satisfies the single crossing property in  $(-r_R, r_S, p_R; -\lambda)$ . Therefore,  $(-r_R, r_S, p_R)$  is non-decreasing in  $-\lambda$ , which concludes the proof.